Hale School **Physics 3A 2010**

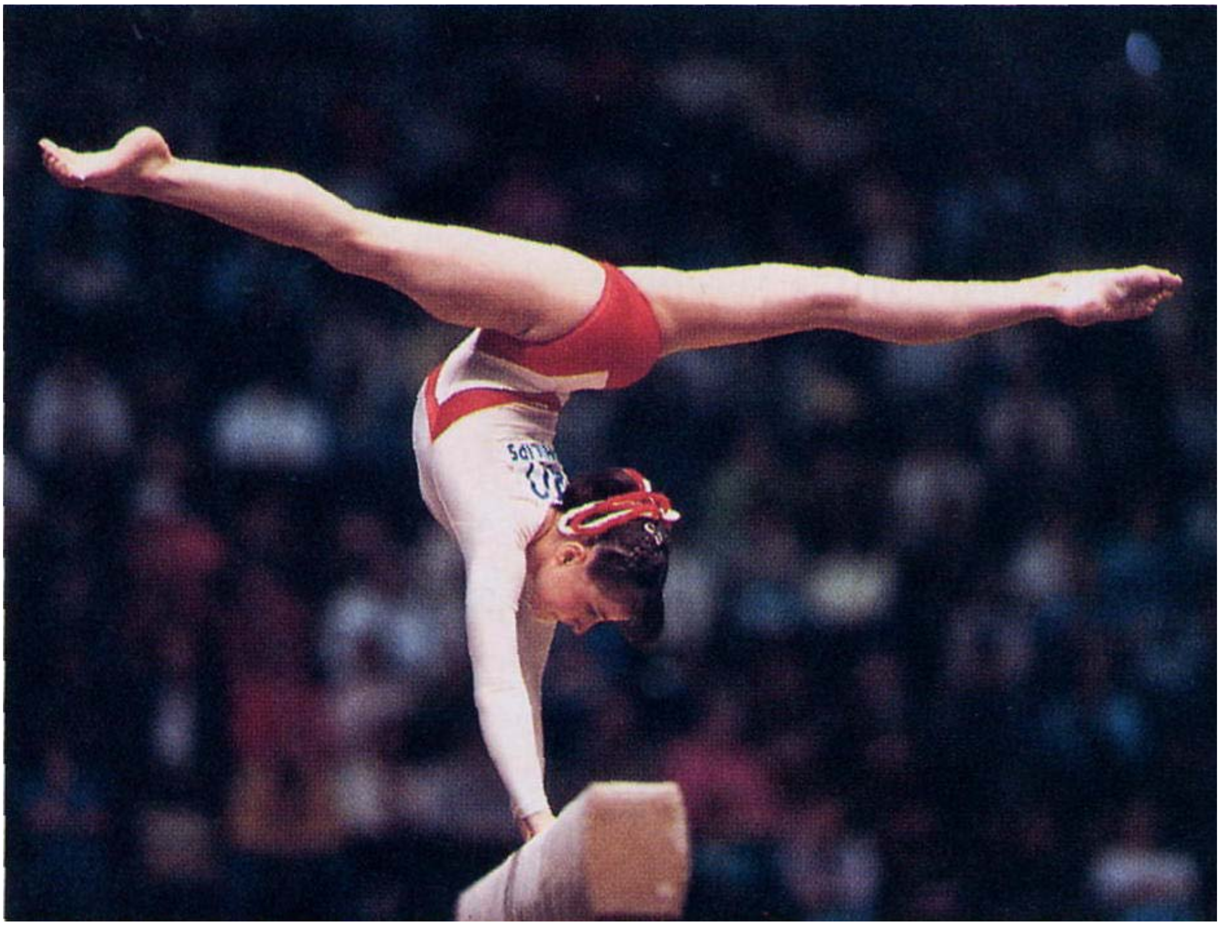
Mechanical Equilibrium

Year 12 Study Notes

Name:

Teacher:

Set:



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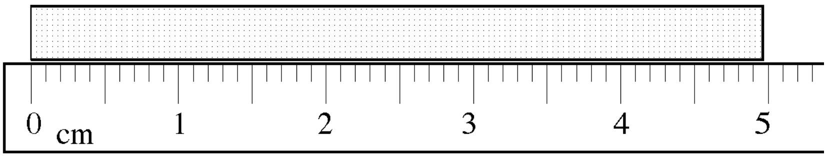
Significant Figures

Significant figures are used to express the degree of precision of a measured value.
Significant figures by definition include all digits known with certainty plus the first uncertain digit.

If a value is recorded as say 23.47, then it implies that the 2,3 and 4 are known to be certain, however the 7 is uncertain. (We are not sure if the 7 is correct.)

When recording values it is important to use the number of digits consistent with the likely uncertainty. In the above example there is no need to record further digits than the 7 as this digit is not certain.

Example A rule is used to measure the length of an object. How should the following measurement be recorded?



We might estimate that the object has a length of 4.96 cm. We are certain of the 4 and the 9 but the 6 is uncertain.

Thus this value has three significant figures. There is no point trying to estimate a more precise reading as our ability to interpolate will probably have an uncertainty of at least ± 0.01cm).

Rules for Significant Figures

- All non-zero digits are significant, (eg 3.46 has 3 significant figures).
- All zeros between non-zero digits are significant (eg. 4.002 has 4 significant figures).
- All zeros to the right of the decimal point following a non zero digit are significant, (eg 0.0500 has 3 significant figures).
- All other zeros are not significant (eg 400 has 1 significant figure, 0.0024 has 2 significant figures)

Addition and Subtraction

When values are added the number of significant figures depends upon the position of the left most uncertain digit.

Example:

	2.47	
	1.425	.... left most uncertain figure
+	0.4	.... answer incorporating the correct number of significant figures.
=	4.295	
=	<u>4.3</u>	

Multiplication and Division

When numbers are multiplied or divided the answer has the same number of significant figures as the term which has the lowest number of significant figures.

Example:

$$\frac{4.24 \times 2.1}{3.692} = 2.411701 = 2.4$$

The term with the lowest number of significant figures is 2.1 Thus the answer is written to 2 significant figures.

Rounding

If the last digit of a number is less than 5 round down

eg 1.24 = 1.2 .... rounded to 2 significant figures

If the last digit of a number is greater than 5 round up

eg 0.2437 = 0.244 .... rounded to 3 significant figures

If the last digit of a number is 5 then round to the even digit

eg 4.355 = 4.36 .... rounded to 3 significant figures

while 2.45 = 2.4 .... rounded to 2 significant figures.

Approximations and Estimate Questions

It is sometimes necessary to make valid approximations of quantities based on "rough" data.

Generally the easiest values to estimate are the fundamental quantities of **M**ass, **L**ength and **T**ime. Everything else can usually be derived from these.

- Clearly state your assumed values with appropriate units and to 1 significant figure.
- Show all important relationships and set out the algorithm in a logical sequence.
- State your answer to 1 significant figure or as an order of magnitude.

Example: Determine the approximate pressure exerted by a coin on a flat horizontal surface.

Reasoning: Mass of coin = approx 10g = 1×10^{-2} kg

Area of coin = say 1 cm^2 = 1×10^{-4} m^2

Thus since $P = \frac{F}{A} = \frac{m\,g}{A} = \frac{10^{-2} \times 10}{10^{-4}}$

Order of magnitude of the pressure = 10^3 Pa

Uncertainty of Measurement

It will be necessary to determine the uncertainty of measurements for experimental work. A number of factors should be considered in determining the uncertainty of a measured quantity. No instrument can measure a quantity exactly and thus there is an *uncertainty* associated with each measurement.

The term uncertainty is preferred to instrumental error as error implies this may be corrected. The uncertainty associated with a measurement is a component of the measurement and should always be taken into account. When recording a measurement the correct number of significant figures and the associated uncertainty should be included.

Scale Reading Uncertainty

The scale reading uncertainty of an instrument is normally defined as *half the smallest division marked on the instrument.*

For example, the smallest subdivision on a standard metre rule is 0.1 cm (1 mm).

Thus the scale reading uncertainty for a metre rule is ± 0.05 cm (0.5 mm).

In most cases it is possible to read an instrument to an accuracy greater than the scale reading uncertainty. This is termed *interpolation* between the graduations. Wherever possible you should interpolate between the graduations to reduce the uncertainty of your reading.

In obtaining the **best estimate of uncertainty**, you are expected to interpolate between the graduations. In almost all cases this will be **less than the scale reading uncertainty**.

Stating Uncertainty

In stating uncertainties, the following conventions should be adopted.

The unit for a measured quantity and its uncertainty should appear after the uncertainty (eg 24.1 ± 0.1 g is correct, 24.1 g ± 0.1 or 24.1 g ± 0.1 g are incorrect).

Although associated uncertainty can be given as an absolute or percentage value, the absolute value is normally used (eg 25 ± 1^{o}C rather than (25 ± 4%)^{o}C).

Uncertainties are normally given to 1 significant figure.

For example, suppose you calculate the density of an object to be 2.6534 gcm^{-3} and calculate its uncertainty as 0.04283 gcm^{-3}, then the value should be recorded as 2.65 ± 0.04 gcm^{-3}.

Adding and Subtracting Values

When adding or subtracting values the **actual uncertainties are added**.

eg 28.6 ± 0.05 g
- 18.2 ± 0.05 g
= 10.4 ± 0.1 g

Multiplying and Dividing Values

When multiplying or dividing values, the **<u>percentage</u> uncertainties are added**.

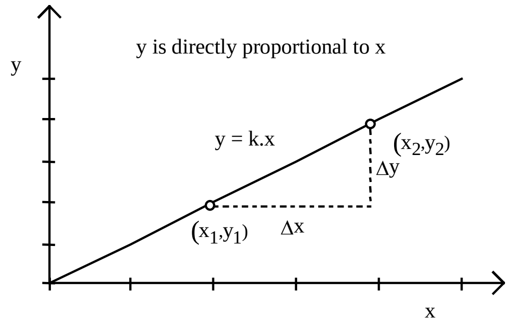
Graphical Analysis

Graphing enables us to determine the relationship between two variables.

One variable (the independent variable) is altered and the value of the other variable (the dependent variable) is measured. Values of the independent variable are plotted on the abscissa (x or horizontal axis) while values of the dependent variable are plotted on the ordinate (y or vertical axis).

Direct Proportionality

A straight line through the origin of a graph indicates a direct proportionality between the two variables.



The straight line graph implies $y \alpha x$

$y = k.x$

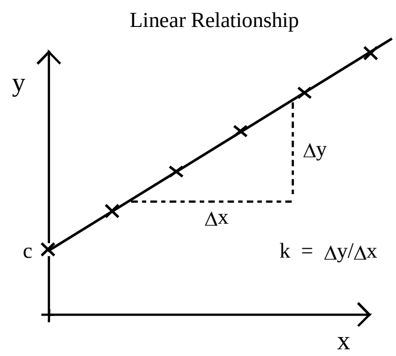
where k is the proportionality constant.
k determines the slope of the graph.
(ie $k = \Delta y/\Delta x$)

Consider two corresponding sets of values x_1y_1 and x_2y_2

$$\frac{y_1}{x_1} = \frac{y_2}{x_2} \quad \text{or} \quad \frac{y_1}{y_2} = \frac{x_1}{x_2}$$

Linear Relationships

A straight line that has an intercept on the ordinate indicates a linear relationship exists between the two variables.

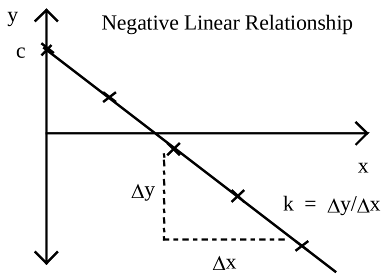


The general equation for the relationship is
y = kx + c

k is the slope or gradient of the line c is the intercept on the y axis.

Negative Slopes

The following graph is produced when **k has a negative value**.

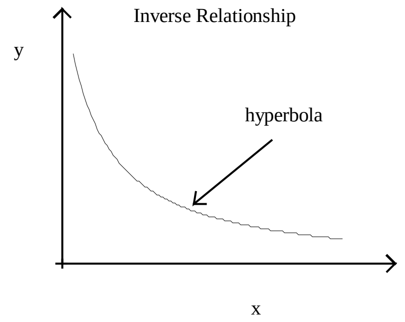


Inverse Relationships (Hyperbolic Relationships)

An inverse relationship between variables, results in a curve called a hyperbola.

The hyperbola indicates

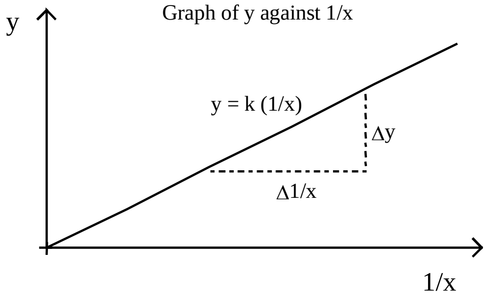
$$\mathbf{y \,\alpha\, (^1/_x)}$$



If an inverse relationship exists and one variable is plotted against the reciprocal of the other, a straight line is obtained.

Since x.y = k for any two corresponding sets of variables then

$$\frac{y_1}{x_1} = \frac{x_2}{y_2} \quad \text{or} \quad \frac{y_1}{y_2} = \frac{x_2}{x_1}$$

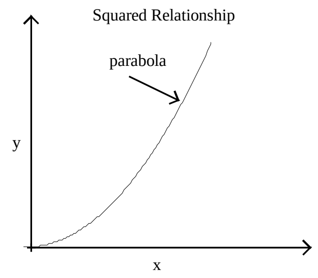


Squared Relationships (Parabolic Relationships)

A squared relationship between variables results in a curve called a parabola.

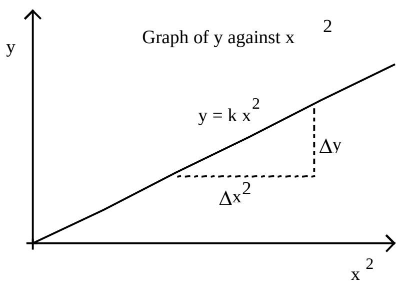
The graph indicates that $y \;\alpha\; x^2$

ie $\mathbf{y = k.x^2}$



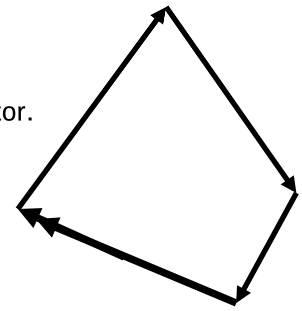
If for the above relationship, y is plotted against x^2, a straight line results.

Since a straight line is obtained when y is plotted against x^2 the relationship between y and x is verified and the value of k can be determined from the slope of the line.



Vector Techniques

Representation of Vectors



- A vector may be represented using a directional arrow.
- The length of the arrow is proportional to the magnitude of the vector.
- The arrow head indicates the direction of the vector.
- A double-headed arrow indicates the vector resultant.

Vector Addition

When two or more vectors interact the “effect” of their interaction can be represented by a single vector that is termed the resultant vector.

The resultant vector is the vector sum of the individual or component vectors.

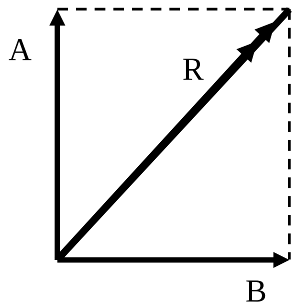
To determine the resultant of a number of individual vectors or component vectors

1. The vectors must be joined “head to tail”. This may occur in any order.
2. The resultant vector is found by joining the “tail of the first vector to the head of the last”.

Adding Vectors at Right Angles

If vectors acting at right angles to each other are added, then the magnitude of the resultant vector is found using Pythagoras’ Theorem.

$$R^2 = A^2 + B^2$$



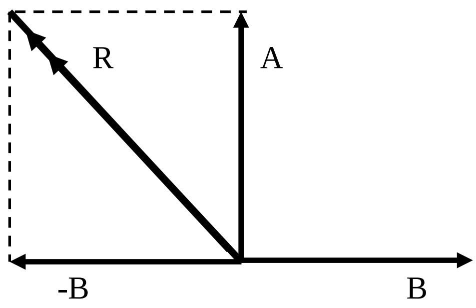
Vectors Subtraction

The negative of a vector has the same magnitude as that vector but opposite direction.

Vector subtraction is carried out as follows:

1. reverse the direction of the vector to be subtracted;
2. then add as normal.

$$R^2 = A^2 + (-B)^2$$

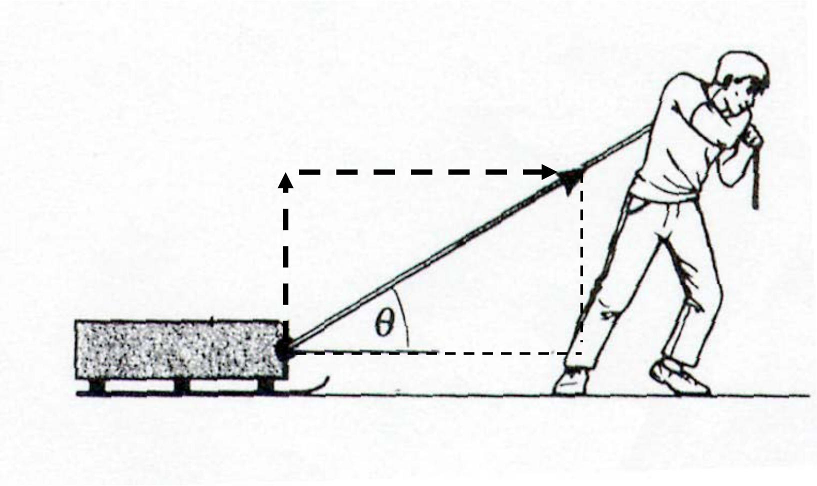


Change in a Vector

This is common example of a vector subtraction. It is often necessary to find the change in the velocity or momentum of a body.

Vector Resolution

The effect of a vector in a given direction may be determined by resolution of the vector in the required direction.



The vertical component of the applied force is:

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The horizontal component of the applied force is:

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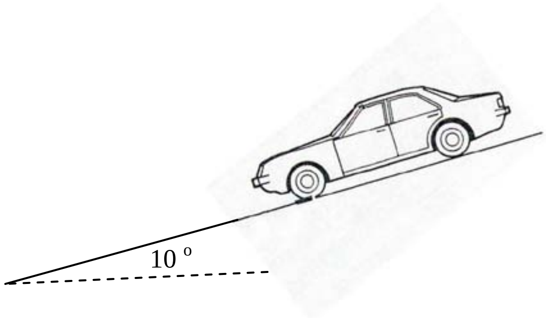
EXAMPLE: Consider the following situation in which a car accelerates down a "smooth" incline....

Estimate the car's acceleration



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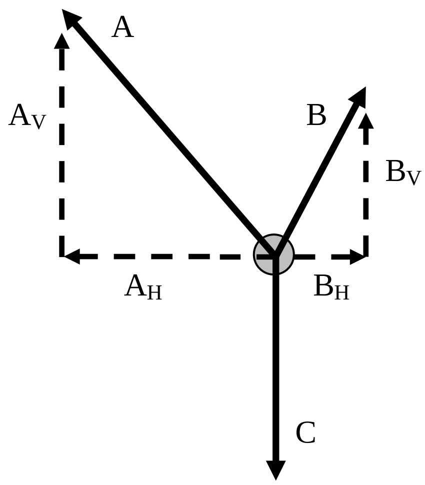
Solving using Resolving

In addition to using sine or cosine rules, vector resolution is a useful way of determining resultants by first resolving into mutually perpendicular components, simplifying parallel components, re-assembling into one pair of perpendicular vectors, and finally recombining.



Independence of Vectors

Consider a boat that is heading due east across a river that is flowing due south.

The boat travels 8 metres east in one second and it will also travel 2 metres south in the same second.

The southerly velocity cannot change the easterly velocity nor can the easterly velocity affect the southerly.

Each is independent of the other and acts as if it were the only velocity on the boat.

Suppose that the river being crossed is 80 metres wide...

It will take the boat 10 seconds to cross the river.

It will also be carried 20 metres downstream during this time.

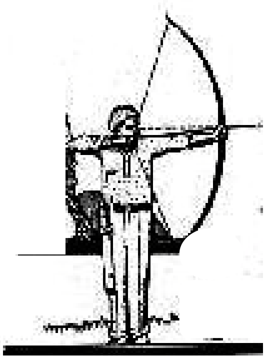
The downstream velocity does not change the river crossing velocity. (ie the vectors are independent)

The resultant vector describes how the boat could arrive at the final outcome (location) under the influence of only a single vector acting on the boat.

(One single velocity vector that has both easterly and southerly velocity components)

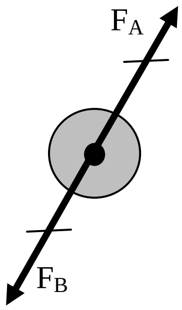
EXAMPLE: Consider the following situation in which an arrow is fired horizontally...

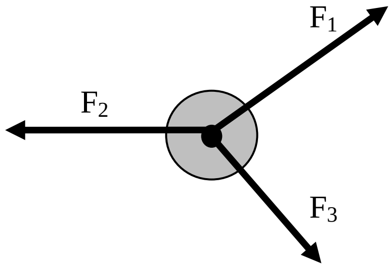
Estimate (ignoring air resistance) how long it is in flight and how far it travels.

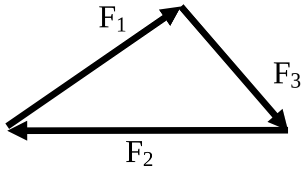


Concurrent Forces

When two or more forces act concurrently on an object and their vector sum is zero, they are "balanced" and the object is in "equilibrium". (mechanical equilibrium will be carefully defined later)







$\mathbf{F_R = F_1 + F_2 + F_3}$

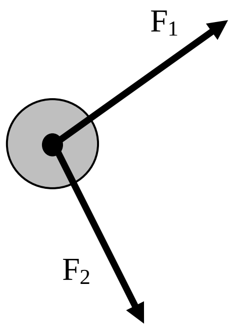
A resultant cannot be drawn when a vector sum is zero!

In such cases simply state the resultant (net) force is zero...

$\boxed{\Sigma \mathbf{F} = 0}$

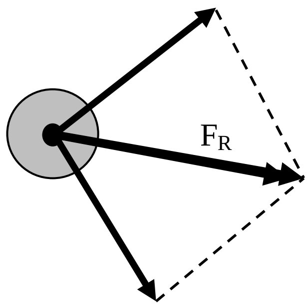
Equilibrant Force

When two or more forces act on a point and their vector sum is NOT zero, an equilibrant force can be determined.

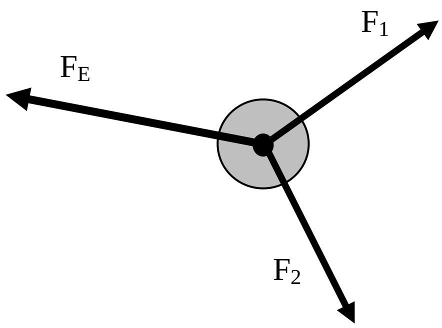


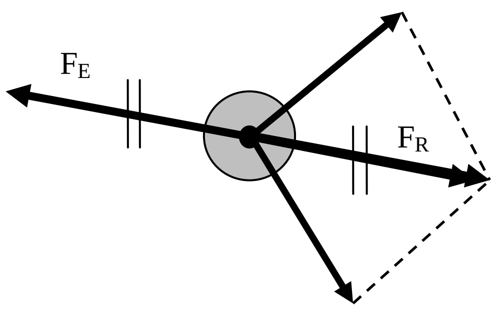
To find the equilibrant force, first find the resultant of the existing forces...

$\mathbf{F_R = (F_1 + F_2)}$



The equilibrant force (F_E) is the single additional force which when applied at the same point as the other forces, results in equilibrium.

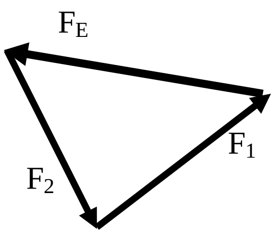




$\mathbf{F_R = (F_1 + F_2)}$

$\mathbf{F_E = - (F_1 + F_2)}$

$\mathbf{F_E = - F_R}$



F_E is the equilibrant to $(F_1 + F_2)$

$\mathbf{F_R = F_1 + F_2 + F_E}$

$\Sigma \mathbf{F} = 0$

Free-Body Diagrams (a problem solving strategy)

When working with problems in which two or more forces or components of force act on a body, it is convenient and instructive to draw a free-body diagram.

In such a diagram we show all the forces acting on a body and if several bodies are involved, we may draw a diagram for each separately.

In illustrations of physical situations (aka space diagrams) force vectors may be drawn at different locations to indicate points of application.

However, since we are concerned only with linear motions, vectors in free-body diagrams may be shown emanating from a common point (centre of gravity) which is chosen as the origin of a x-y axes.

One of the axes is generally chosen along the direction of the net force on the body as that will be the direction of the resulting acceleration.

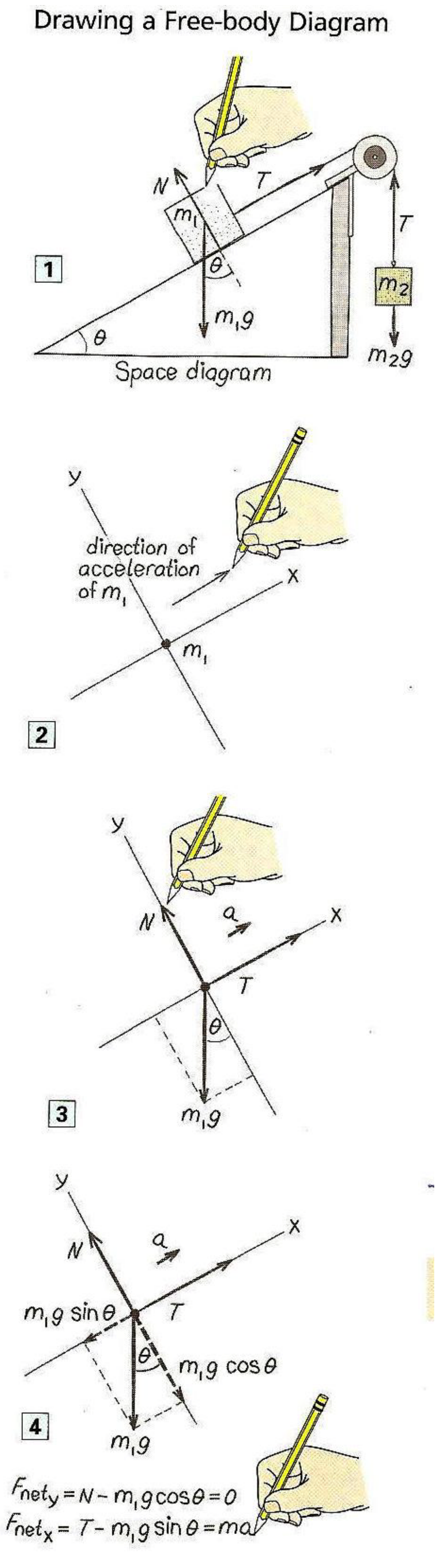
Also it is often important to resolve forces into components, and a properly chosen x-y axes simplifies this.

The vector arrows do not have to be drawn exactly to scale, but it should be made apparent if there is a net force and whether forces balance each other in a particular direction.

General Steps for drawing diagrams

1. Sketch a space diagram (if one is not already given) and identify the forces on each body of the system.
2. Isolate the body of interest and draw a set of axes such that the origin is the centre of gravity or the intersection of relevant lines of action of the forces.
3. Draw properly oriented force vectors emanating from the origin such that they conveniently relate to the net force (or acceleration) of the body.
4. Resolve any forces that are not directed along the axes into (convenient) mutually perpendicular components.

The visualisation of the physical situation into clearly related force vectors will greatly assist the development of efficient problem solving strategies.



Free-body Diagrams

-are vector diagrams which show all of the significant forces acting on a body in a certain situation.

- Only the forces acting on one particular object is the focus of the diagram
- All vectors are drawn from the objects centre of gravity (at least to the extent possible)
- Points of particular reference such as pivot points should be clearly identified.
- All vectors should be clearly labelled and reveal key relationships such as components
- Useful component vectors should be illustrated as dotted lines (to avoid confusion)
- A “line of action” for the forces being considered may be useful.
- Avoid the possible confusion with action/reaction pairs (which object?)
- It is advisable to construct these diagrams (large) with ruled lines.

Exercise: Construct a free body diagram for each of the following situations…

A. a boat sailing in the ocean at a constant speed.

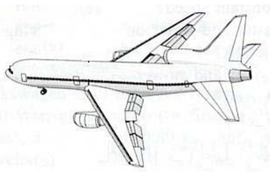
B. a jet accelerating in a straight horizontal direction.

C. a car at rest (parked) on a banked road.

D. a car accelerating uphill along a straight incline.

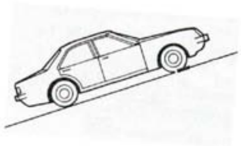
A

B



C

D



Exercise Set 1: Vector techniques (revision)

1. Use scale diagrams to add the following vectors, and state the magnitude and direction of the resultant vectors. a. 15 m north and 20m west b. 300 m N45°W and 400 m S45°W

2. Use Pythagoras' theorem to add the following pairs of vectors. Describe the resultant vectors.
   a. 6 m south and 8 m west b. 7 m S60°E and 24 m S30°W

3. Find the resultant displacement in each case of a cyclist who rides
   a. 13 km South then 9 km North. b. 15 km NE then 15 km NW.

4. An aeroplane travelling at 173.2 kmh^{-1} West changes course to 100 kmh^{-1} South. What is the change in velocity of the aeroplane?

5. A car travelling at 200 kmh^{-1}, S 60^o E has a collision after which it travels at 100 kmh^{-1} South. What is its change in velocity?

6. A force of 740 N is applied horizontally to drag a cart up a 30.0^o incline. What are the components of this force up the incline and perpendicular to it?

7. A gun is fired at an angle of 30° to the horizontal. If the vertical component of the bullet's velocity on release is 75 ms^{-1} , calculate
   a. the actual speed of the bullet b. the horizontal component of the velocity of the bullet.

8. Two boys are dragging an object along level ground by pulling on a rope held horizontally with forces of 150 N and 180 N respectively.
   a. What single force would have the same effect?
   b. If the frictional force is 30 N, what is the resultant horizontal force on the object?

9. Two tugs are pulling on a ship, one with a force of 6.0 kN and the other with a force of 8.0 kN. The cables from the tug to the ship are at right angles to each other. Find the magnitude of the resultant force on the ship, and the angle that its direction makes with the cable in which the force is 8.0 kN.

10. A cyclist travelling at 12 km h^{-1} east makes a right-hand turn at an intersection without changing speed. What is his change in velocity?

11. A man pushes a garden roller weighing 1000 N over a level lawn, holding the end of a 150 cm long handle 90 cm above the axis of the roller as shown in Figure 8.3.

    He pushes down the handle with a force of 200 N.

    a. What is the size of the effective force pushing the roller along?

    b. With what vertical force does the lawn press against the roller while the man is pushing the handle?

    c. What vertical force would the lawn exert on the roller if the man pulled it (with the same force) instead of pushing it, the handle remaining in the same position.

    d. If the roller lodges in a small rut, would it be easier to get it out by pulling or by pushing?

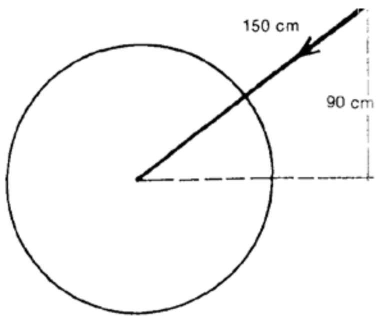
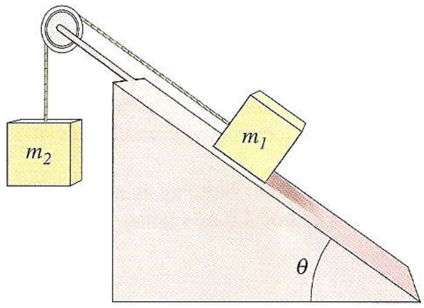


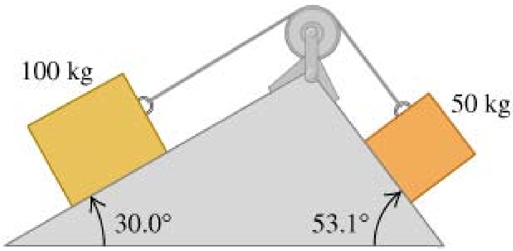
Fig. 8.3

12. Construct a free-body diagram for each of the following physical situations: (assume that the system is accelerating in both cases)

A.



B.



ANSWERS

1. a. 25 m N53.1°W b. 500 m S82°W
2. a. l0 m S53.1°W b. 25 m S13.7°W
3. a. 4 km South, b. 21.2 km N
4. 200 kmh^{-1}, S60°E
5. 173 kmh^{-1} West
6. 641 N, 370 N
7. a. 150 ms^{-1} b. 130 ms^{-1}
8. a. A pull of 330 N b. 300 N in the direction of the pull
9. 55 m s^{-1} north
10. 17 km h^{-1} south-west
11. a. 1.6 x 10^{2}N b. 1.1 x 1O^{3} N c. 8.8 x lO^{2} N d. Pulling

Mechanical Equilibrium

Thus far in our study of forces, we have been treating all the forces acting on an object as if they were acting at a single point.

However, there may be several forces, each acting at a different point on the object.

When considering the effect a force has on the motion of a body it is necessary to know:

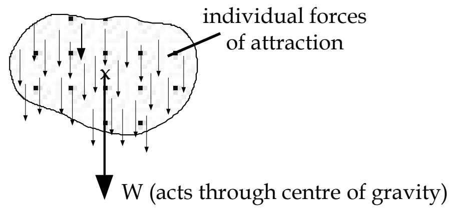
- its shape
- its mass distribution
- where the forces act

When examining the mass distribution of a body, it is important to relate to the “centre of mass” and “centre of gravity” of the body.

The entire mass of a body can be considered to act at its “centre of mass”.

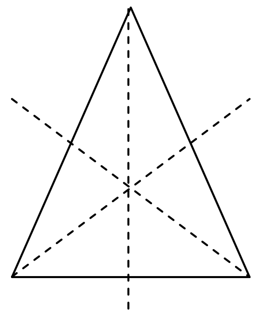
The concept of the “centre of gravity” is used to determine the “centre of mass” of an object.

Centre of Gravity



A body may be considered as consisting of a very large number of particles, each of which is attracted towards the Earth's centre.

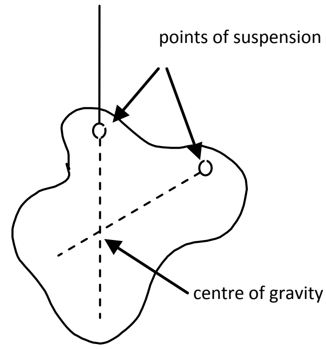
The resultant of all these individual forces is a single force termed the body’s weight (W) that acts vertically down through a single reference point.



The single point through which the weight of a body may be considered to act is called its “centre of gravity”.

The centre of gravity is the point at which the whole weight of the body can be taken as acting no matter in what position the body is placed.

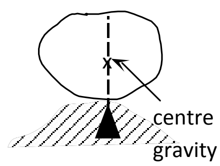
The centre of gravity of “regular” and “uniform” objects (all made of the same material), will be located at their geometric centres.



The centre of gravity of “irregular” or “non-uniform” objects must be found experimentally. The centre of gravity may be located by freely suspending the object from a single point.

A vertical line through the point of suspension will pass through the centre of gravity.

The centre of gravity may be regarded as the point of balance.

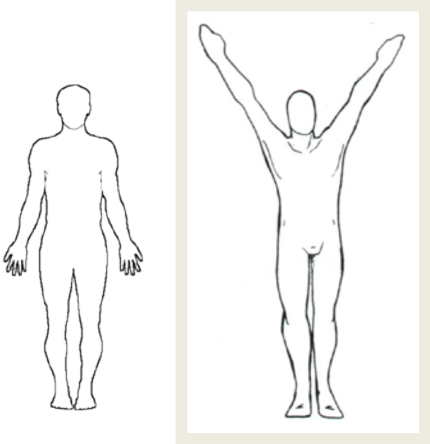


single upward force through the centre of gravity

A body may be supported by a single upward vertical force if the force is equal to the body’s weight and passes through the centre of gravity of the body.

The balancing rock remains “stable” as long as the point of contact is vertically below the centre of gravity.

Centre of Mass



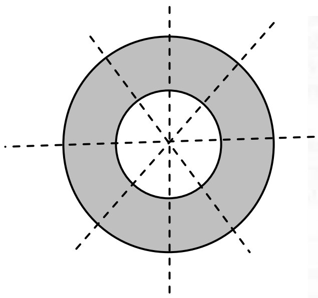
The mass of a body is distributed throughout body.

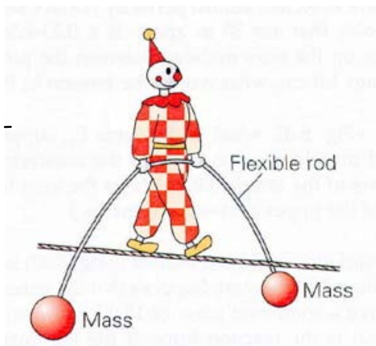
However for practical purposes we can consider the mass to act through a single reference point called the "centre of mass".

The "centre of mass" of a body coincides with its "centre of gravity" assuming no variation in gravitational attraction throughout the body.

The position of our centre of gravity can vary according to the human body's shape. The configuration of arms and legs will change the distribution of a body's mass and hence, its centre of gravity.

Some objects have an external centre of gravity.
(i.e not found on the object itself)

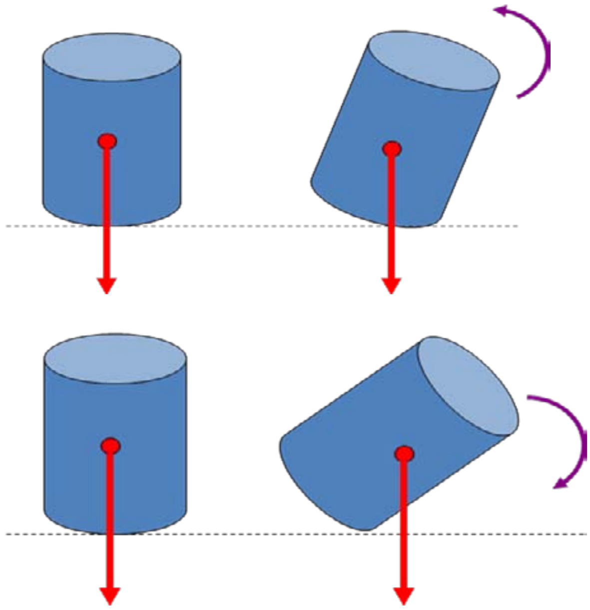




The high jumper's centre of mass lies outside of (below) his body as she clears the bar.

The centre of mass follows a trajectory below the bar.

Stable Equilibrium



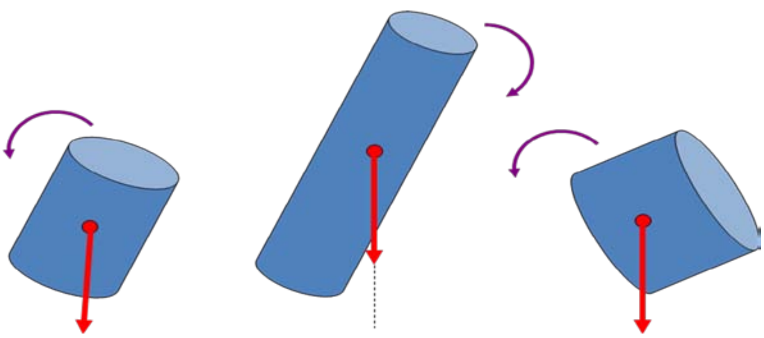
A body is in stable equilibrium when it returns to its original position after being given a small displacement.

A cylinder standing on its base is in stable equilibrium.

A vertical line ("plumb-line") from the centre of gravity falls inside the base and so the object's weight returns it to its original position.

When the vertical from the centre of gravity falls outside the base, the body topples over.

The lower the centre of gravity, the further the body can be displaced and still return to its stable position when the displacing force is removed.



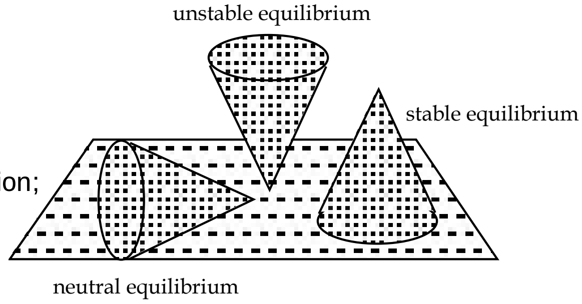
Stability and Toppling

If given a small displacement, a body in :

stable equilibrium returns to its original position;

unstable equilibrium moves away from its original position;

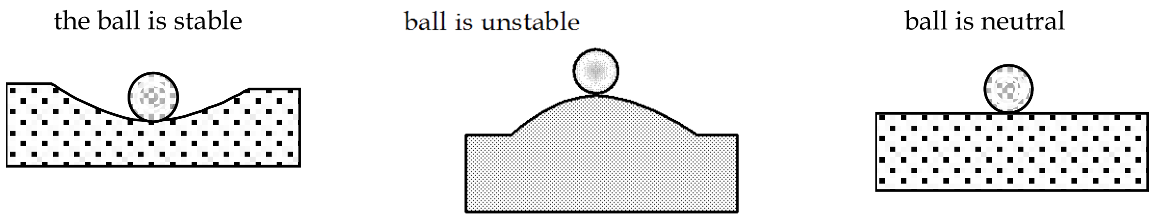
neutral equilibrium remains in its displaced position



Stability and Centre of Gravity

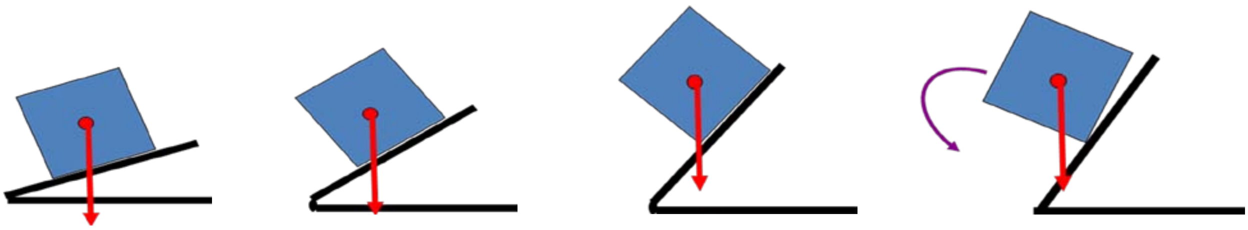
If a small sideways displacement of the body results in:

- an **increase** in the height of the centre of gravity, the body is in **stable** equilibrium;
- a **decrease** in the height of the centre of gravity, the body is in **unstable** equilibrium;
- **no change** in the height of the centre of gravity, the body is in **neutra**l equilibrium.

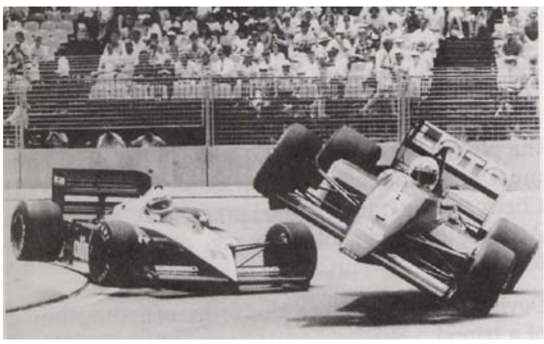


Toppling -Overturning

A body will not overturn ("topple") so long as a vertical line through the centre of gravity of the body passes through its base.



To minimize the tendency of a body to overturn, the centre of gravity should remain low and the base should be as wide as possible.



Show the c.o.g and "plumb line" in each case...

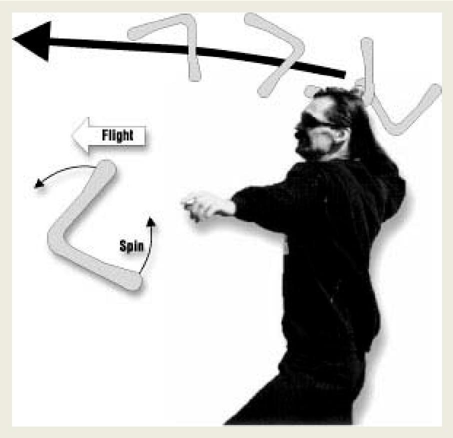
Translational and Rotational Equilibrium

When a force (or forces) acts on a body it may result in a change in the body's translational and/or rotational motion.

Translational Motion: Motion in a straight line.

Rotational Motion: Motion about an axis.

For example, when a boomerang is thrown it has both translational motion due to its forward movement and rotational motion due to its spinning movement.



Mechanical Equilibrium occurs when the forces that act on a body have no effect on either its translational or rotational motion.

If a body is at rest or moving with uniform velocity, it is said to be in equilibrium.

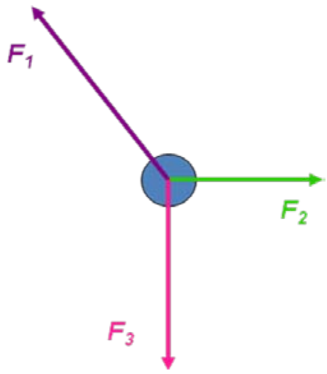
For a body to be in mechanical equilibrium:

1. the vector resultant of all the forces acting on it must be zero: (Translational equilibrium) $\boxed{\Sigma \mathbf{F} = 0}$

and

2. the vector sum of the torques (or moments) acting on the body must be zero: (Rotational equilibrium) $\boxed{\Sigma \mathbf{M} = 0}$

Translational Equilibrium - Concurrent Coplanar Forces

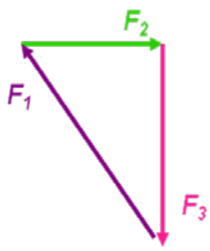


Forces are concurrent if their lines of action intersect at one point.

Consider 3 concurrent coplanar forces acting on a body:

For translational equilibrium: $\Sigma \mathbf{F} = 0$

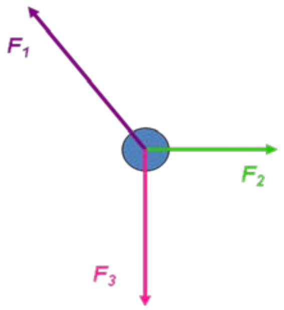
$$\Sigma \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \text{zero}$$

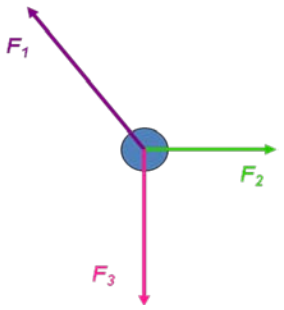


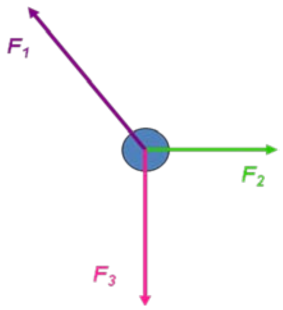
thus the forces joined head to tail must form a closed triangle and the resultant force is zero.

Note that each force is the **equilibrant** of the sum of the other two.

EXERCISE: Carefully illustrate the equilibrant for each force on the vector diagrams below:



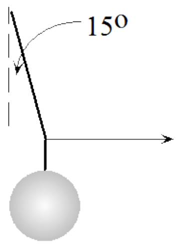


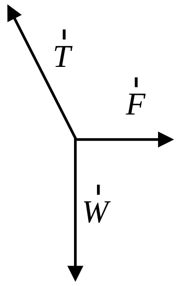


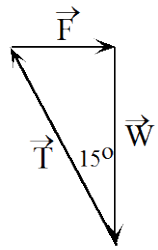
Solving Equilibrium Problems:

- Draw a free body diagram (also called a space diagram) of the object (or point) in equilibrium.
- Mark each of the forces with a vector arrow in the correct direction and indicate the angles.
- Mark only the forces applied to the object or point. Forces applied by this object to other things are irrelevant.
- Draw a closed vector triangle, arranging the forces "head to tail". Show all known forces and angles between them. The unknown force will be one side of the triangle.

Example 1:
A metal sphere of mass 200.0 kg connected is pulled horizontally sideways until the cable makes an angle of 15.0^0 with the vertical. Find the horizontal force required and the tension in the cable.







$$\sum \vec{F} = 0$$

i.e. $\vec{F} + \vec{W} + \vec{T} = 0$

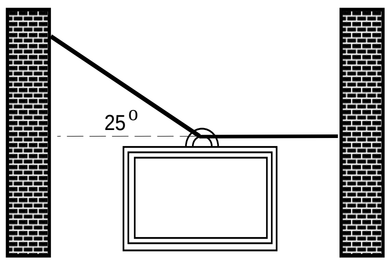
F = W $\tan 15^0$ = mg $\tan 15^0$

F = 200 x 9.8 x $\tan 15^0$ = **525 N**

T = $\dfrac{W}{\cos 15}$ = $\dfrac{mg}{\cos 15}$

T = $\dfrac{200 \times 9.8}{\cos 15}$ = **2.03×10^3 N**

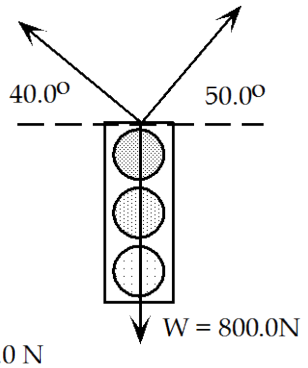
Problem 1: A sign of mass 50.0 kg is supported by cables as shown. Find the tension in each cable if one cable is horizontal and the other makes an angle of 25.0^0 to the horizontal.



Example 2:

A set of traffic lights weighing 800.0 N is suspended above a roadway by two cables which make angles of 50.0^0 and 40.0^0 to the horizontal.

Find the tension in each cable.



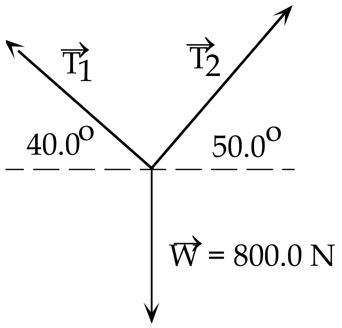
W = 800.0 N

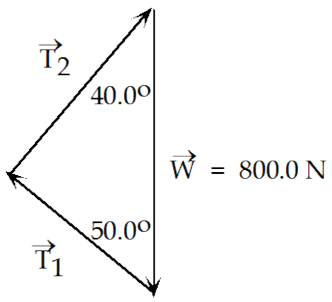
$\mathbf{T_1}$ = W $\sin 40^0$

$\mathbf{T_1}$ = **<u>514 N</u>**

$\mathbf{T_2}$ = W $\cos 40^0$

$\mathbf{T_2}$ = **<u>613 N</u>**



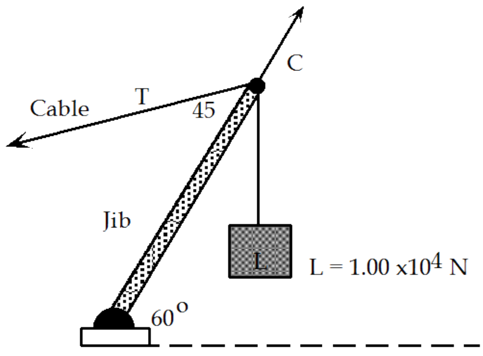


<u>Note</u>: For more than 3 forces you must resolve into rectangular components.

Example 3:

A load weighing 1.00 x 10^4 N is supported as shown.

Determine the tensile force in the cable and the compression force in the jib.



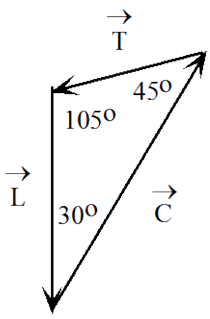
Solution:
The diagram we are given shows all the forces acting on the point that is in equilibrium.

This is the point at the top of the jib.

The vector labelled *C* is a reaction force from the top of the jib.

The jib itself will have a strong force of compression acting along its length. But we ignore this force as it is irrelevant. We are only interested in forces acting **on** the point at the top of the jib, not the compressive force it is applying to the jib.

$\sum \vec{F} = 0 \quad \therefore \vec{L} + \vec{T} + \vec{C} = 0$



Using the sine rule $\frac{T}{\sin 30}$ $\quad \therefore \quad T = \frac{L \sin 30^0}{\sin 45^0}$

$\therefore \quad T = \frac{10000 \times \sin 30^0}{\sin 45^0}$

$\therefore \quad T =$ **7.07 kN**

and since $\frac{C}{\sin 105} = \frac{L}{\sin 45}$

$\therefore \quad C = \frac{\sin 105 \times 10000}{\sin 45}$

$\therefore \quad C =$ **13.7 kN**

Alternative Solution:

We can resolve all forces into components that are **perpendicular** and **parallel** to the jib.

Forces **perpendicular** to the jib: $T \times \cos 45^0 = L \times \cos 60^0$

$$\therefore \quad T = \frac{L \cos 60^0}{\cos 45^0} = 7.071 \times 10^3 \text{ N} \quad (7.07 \times 10^3 \text{ to 3 s.f.})$$

Forces **parallel** to the jib: Forces acting 'up' the jib will equal forces acting 'down' the jib.

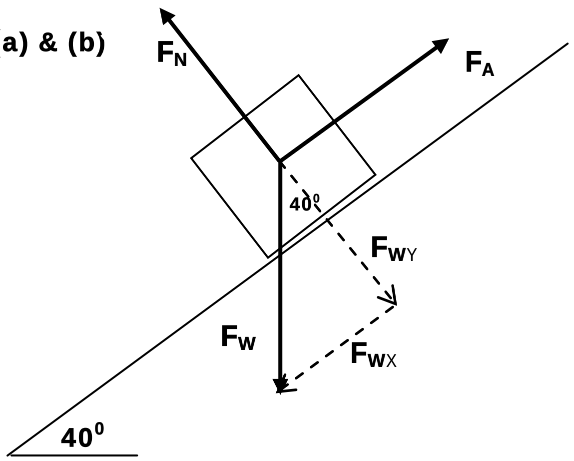
$C = L \times \sin 60^0 + T \times \cos 45^0 = 10{,}000 \times \sin 60^0 + 7071 \times \cos 45^0$

$C = 1.37 \times 10^4$ N

Example 4:

A crate of mass 50.0 kg rests on a **smooth ramp** inclined at 40^0 to the horizontal.

(a) & (b)

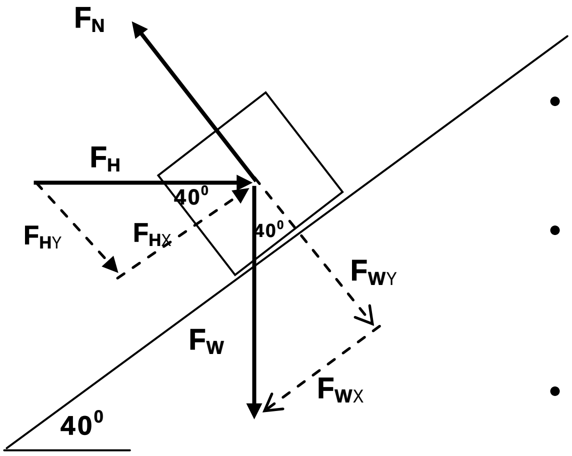


a) What is the minimum force parallel to the ramp required to stop the crate from moving down the slope?

b) What is the reaction force exerted by the ramp on the crate?

c) What is the minimum horizontal force necessary to stop the crate moving down the ramp?

(c)



- The crate is at rest, so forces are balanced. This means forces up the incline equal forces down the incline.

  Forces perpendicular to the incline are also balanced.

- We do not use normal x-y axes. Instead we choose the x-axis to be parallel to the plane and the y-axis to be perpendicular to the plane.

- All vectors must be resolved into components that are parallel and perpendicular to the incline. This means resolving both the weight force F_W and the horizontal force F_H.

- The ramp is smooth therefore friction is not involved.

- The applied force F_A and the horizontal force F_H do not act at the same time. They are different parts of the problem.

a)

$F_A = F_{WX}$

$F_A = F_W \times \sin 40^0$

$F_A = m \times g \times \sin 40^0$

$F_A = 50 \times 9.8 \times \sin 40^0$

$F_A =$ **315 N**

b)

$F_N = F_{WY}$

$F_N = F_W \times \cos 40^0$

$F_N = m \times g \times \cos 40^0$

$F_N = 50 \times 9.8 \times \cos 40^0$

$F_N =$ **375 N**

c) Equate forces parallel to the incline.

$\therefore F_{HX} = F_{WX}$

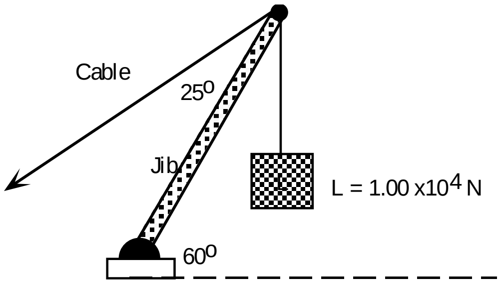
$F_H \times \cos 40^0 = m \times g \times \sin 40^0$

$$\therefore F_H = \frac{50 \times 9.8 \times \sin 40^0}{\cos 40^0}$$

$\therefore F_H =$ **411 N**

Problem 2: The angle that the cable makes with the jib is altered to 25.0°.

Determine: a) the force in the cable
b) the force in the jib



Exercise Set 2: Concurrent Coplanar Forces

1. Which of the following pairs of forces are in equilibrium?
   a) 4.00 N east and 4.00 N west
   b) 2.00 N west and 2.00 N south
   c) 1.50 N west and -1.50 N west.

2. Use diagrams to show that the following forces are in equilibrium:
   a) F_l = 4.00 N west, F_2 = 3.00 N south, F_3 = 5.00 N E37°N
   b) F_1 = 70.0 N east, F_2 = 240 N south, F_3 = 250 N N16°W

3. An object has the following three forces acting on it simultaneously:
   F_l = 2.00 N west, F_2 = 3.00 N south, and F_3 = 6.00 N east.
   a) Determine the resultant force on the object.
   b) What additional force would be required to put the object in equilibrium?

4. a) Lisa, Mario and Amelia are all pulling on a metal ring which is free to move. Lisa pulls north with a force of 50.0 N and Mario pulls west with a force of 120 N. If the ring does not move, determine the force Amelia applies.

   b) A body which is in equilibrium has three forces acting on it. Two of the forces are 3.00 N south and 4.00 N east. What is the value of the third force?

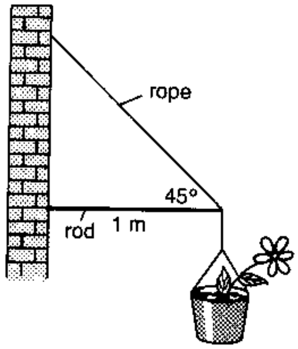
5 A picture that weighs 5 kg (49.0 N) is hung by two wires inclined at 90.0° to each other. Determine the tension in the wires. (Hint: it is the same in each .)

6. A light globe of mass 200 g hangs on the end of a thin cable fixed to the ceiling. A second thin cable fixed to the globe pulls the globe horizontally so that it is at an angle of 30.0^0 to the vertical. Determine the tension in both cables.

7. A pot-plant is suspended from a wall by means of a 1 m rod and a rope.

   If the pot-plant exerts a vertical force of 200N, calculate the tension in the rope.

   (assume the rod exerts a force horizontal away from the wall)



8. A ramp is inclined at 30.0° to the horizontal.
   What frictional force is needed to keep a 100 g block at rest on an incline?

9. A man holds a rope which hangs vertically and supports a mass of 5.00 kg.
   a) What is the tension in the rope?
   b) What forces is exerted by the man on the rope?

10. A bag of sand weighing 100 N is suspended by a long, light rope, but is pulled aside by a horizontal force so that the rope makes an angle of 25.0^0 with the vertical.
    Find the magnitudes of the horizontal force and the tensile force in the rope.

11. Three cords, A, B and C, are attached to one another at a point O.
    The angle between cords A and B is 80^0 and that between B and C is 150^0.

    If the force exerted by cord A is 100 N, determine the values of the forces exerted by cords B and C, assuming the three forces to be in equilibrium.

12. A small barrel weighing 600 N is held at rest on a smooth ramp inclined at 30.0^0 to the horizontal by means of a force parallel to the ramp.
    Find the magnitude of this force and also the reaction of the ramp on the barrel.
    What force would be required if it were applied in a horizontal direction?

13. A painting of mass 1.25 kg is suspended from a single wall hook by a wire of length 96.0 cm attached at two points on the frame separated by 90.0 cm. What is the tension in the wire?

14. Two men carry a mass of 30.0 kg between them by means of two ropes fixed to the mass.
    One rope is inclined at 30.0^0 to the vertical and the other at 45.0^0.
    Determine the tension in each rope.

15. A pendulum bob is hung on a string 1.00 m long and is displaced by a horizontal force so that the string an angle of 45.0^0 with the vertical. If the mass of the bob is 20.0 g, what is the horizontal force required and how much work is done in giving the bob this displacement?

16. Find the horizontal force required to keep a small box of mass 200 g at rest on a smooth plane inclined at 30.0^0 to the horizontal.

17. Three coplanar, concurrent forces of 45.0 N, 75.0 N and 60.0 N are in equilibrium.
    What are the angles between their lines of action?

18. The vertical member of a jib crane is 3.00 m long. The jib has a length of 6.00 m and the length of the tie rod is 5.00 m. The compressive load in the jib is restricted to 50.0 kN. With the aid of a vector diagram drawn to scale, determine the maximum mass, in tonnes, that can be lifted by the crane and the corresponding tension in the tie rod.

ANSWERS

1. A,C

3. **a**) 5 N S53°E **b**) 5 N N53°W

4. **a**) 130 N **b**) 5 N

5. 34.6 N

6. horizontal wire: 1.13 N, vertical: 2.26 N

7. 283 N (Hint: the tension in the rope is along the rope towards the wall while the rod exerts a force away from the wall)

8. 0.49 N (Hint: find the component of the weight acting down the incline.)

9. Both answers 49 N

10. 46.6 N; 110 N

11. 153.2 N, 197 N

12. 300 N, 520 N, 346 N

13. 17.6 N

14. 216 N, 153 N

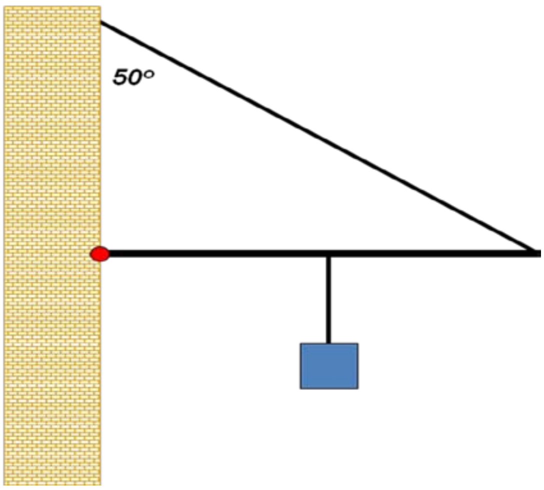
15. 2.00×10^{-1} N, 5.86×10^{-2} J

16. 1.13 N

17. $53^o\,8'$; $36^o\,52'$; 90^o

18. 2.55 t, 41.7 kN

EXERCISE: Consider the Boom-Tie cantilever System shown:



On the diagram provided:

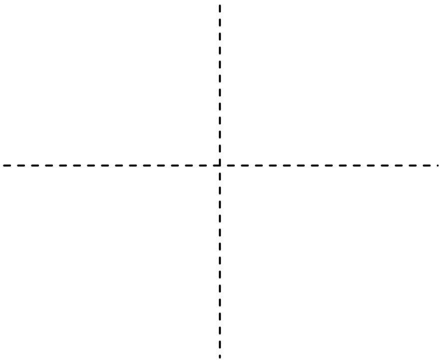
1: Show the main forces acting on the beam.

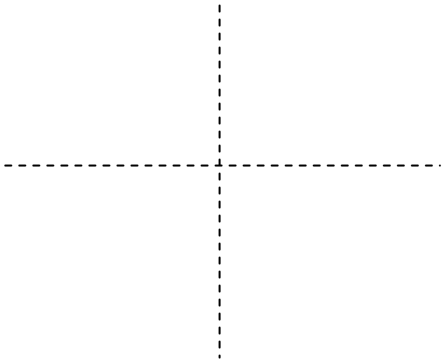
2: Show the lines of action for each force

3: Construct a free-body diagram for the forces in translational equilibrium for the beam. (in the reference grid below left)

4: Draw a Vector diagram for the sum of the forces on the beam. (below)

5: Show and label the vertical and horizontal components of the forces involved (in 4).





Rotational Equilibrium – Moments / Torques

If the line of action of an applied force passes outside the centre of mass it will cause a turning effect.

The magnitude of this turning effect is called the **moment** or **torque** of the force.

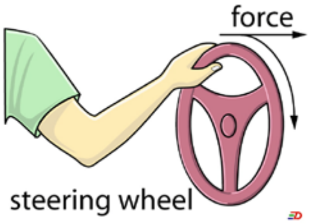
The word ‘Moment’ can be interchanged with the word ‘Torque’.

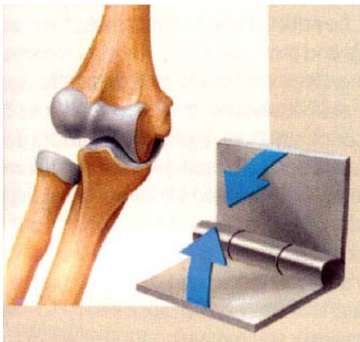
Moment of a Force

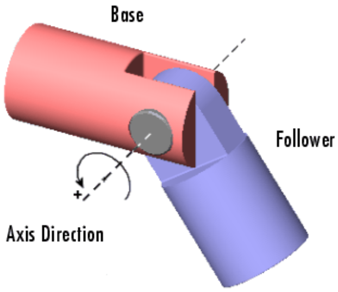
The moment of a force is the product of the force and the radial distance (r).

The radial distance is the perpendicular distance from the point to the line of action of the force.

△ To provide torque, a point of rotation such as a pivot, axel or hinge is always needed (aka “Fulcrum”)





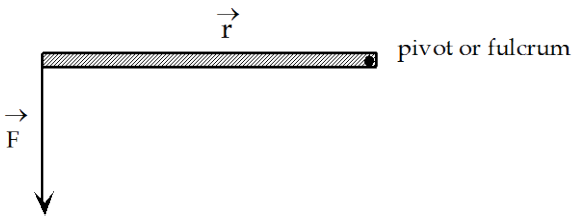


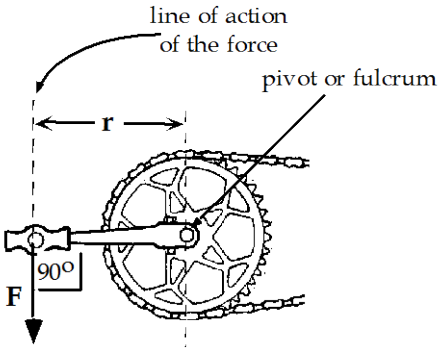
Torque = magnitude of **Force** x perpendicular **Distance**

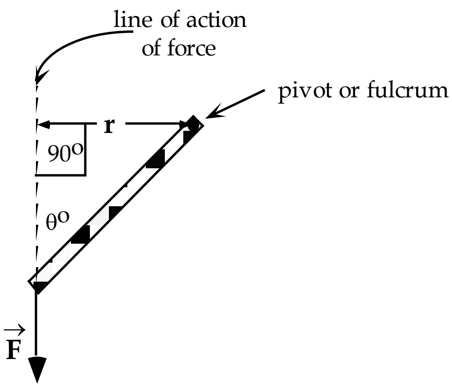
$$\tau = M = F \times r \times \sin\theta$$

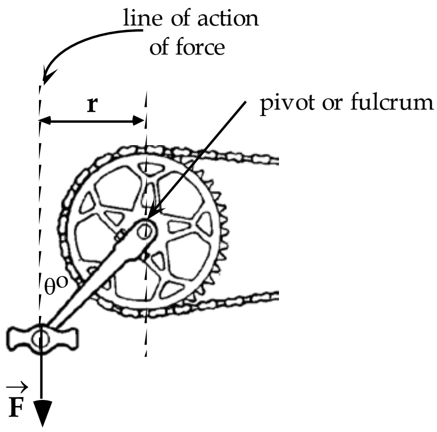
Torque is a vector quantity measured in Newton meters

Two examples:

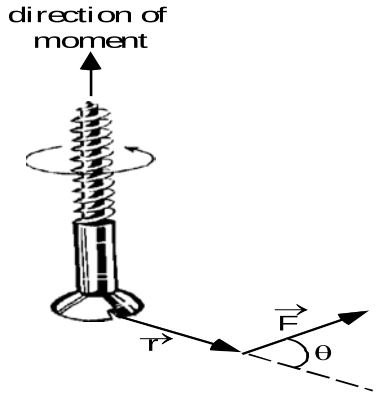








Principle of Moments



A moment is a vector.

The direction of a moment is mutually perpendicular to both $\vec{r}$ and $\vec{F}$.

The right hand screw rule may be used to determine this direction.

However for our work we may consider a moment producing either clockwise rotation or anticlockwise rotation.

A moment producing clockwise rotation acts in the opposite direction to a moment producing anticlockwise rotation.

If a body is in **rotational equilibrium** under the action of a number of coplanar forces, then the sum of the anticlockwise moments is equal to the sum of the clockwise moments.

$$\sum M = 0$$

This is known as the **"Principle of Moments".**

The Principle of Moments is usually expressed as:

$$\sum CM = \sum ACM$$

[Note any point may be selected about which to take moments.]

Moments Applications involving Parallel Forces

BEAMS

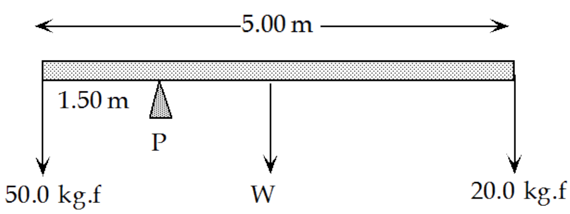
The weight of a beam can be considered to act through its centre of gravity.

For a uniform beam, the centre of gravity is at its mid point.

For the beam to be in equilibrium, the forces that act on it, including its weight, must not produce a resultant moment i.e. $\sum M = 0$.

Example 1:

A uniform beam is 5.00 m long and has a mass of 20.0 kg at one end and 50.0 kg mass at the other. If the beam balances 1.50 m from the 50.0 kg mass, determine the mass of the beam.



Take moments about P:

$$\sum ACM = \sum CM$$

$$\therefore \quad 50 \times 1.5 = W \times 1 + 3.5 \times 20$$

$$W = 75 - 70$$

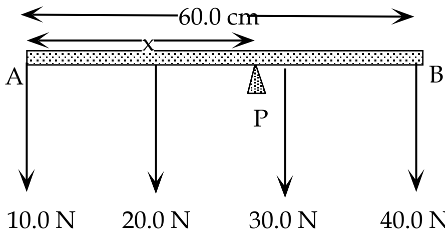
$$\therefore \quad W = 5.00 \text{ kg.f}$$

∴ Mass of the beam = **5.00 kg**.

Example 2:

Weights of 10.0 N, 20.0 N, 30.0 N and 40.0 N are hung at equal distances apart on a beam 60.0 cm long of negligible weight.

Find the point about which the beam will balance.



Take moments about P:

Σ ACM = Σ CM

$\therefore 10(x) + 20(x - 20) = 30(40 - x) + 40(60 - x)$

$\therefore 100(x) = 400 + 1200 + 2400$

$\therefore x =$ <u>40 cm</u>

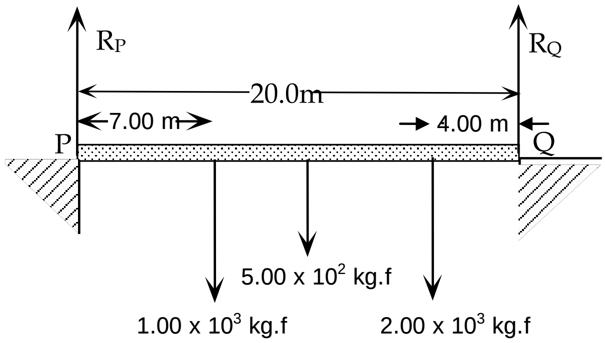
Beam balances at 40.0 cm from end A

Example 3:

A uniform beam PQ is supported at each end.

The beam is 20.0m long and weighs 500.0 kgf.

Loads of 1.00×10^3 kgf and 2.00×10^3 kgf are located 7.00m and 4.00m from ends P and Q respectively.



Determine the reaction forces R_P and R_Q.

Solution:

Take moments about P:

Σ ACM = Σ CM

$\therefore 20\,R_Q = 7(1000) + 10(500) + 16(2000)$

$20\,R_Q = 44000$

$\therefore R_Q = 2200$ kg.f

$\therefore R_Q =$ **<u>2.16×10^4 N</u>**

Take moments about Q:

Σ ACM = Σ CM

$\therefore 20\,R_P = 13(1000) + 10(500) + 4(2000)$

$\therefore 20\,R_P = 26000$

$\therefore R_P = 1300$ kg.f

$\therefore R_P =$ **<u>1.27×10^4 N</u>**

Alternative Solution:

Having determined R_Q using the principle of moments, we may also assume the vector sum of all forces acting on the beam is zero.

i.e. $\sum \vec{F} = 0$ or $\vec{F}_{up} = \vec{F}_{down}$.

$\therefore R_P + R_Q = 1000 + 500 + 2000$

$R_P = 3500 - 2200$

$= 1300$ kg.f

$=$ **<u>1.27×10^4 N</u>**

Exercise Set 3: Principle of Moments

1. When only the front wheels of a car are run onto a platform scale, the scale reads 8.00 kN; and it registers 6.00 kN when only the rear wheels are run onto the platform scale. The distance between axles is 2.80 m.
   a) What is the weight of the car?
   b) How far is its centre of gravity behind the front axle?

2. Weights of 8.00, 5.00, 3.00 and 10.0 N are located, at 25.0, 50.0, 75.0 and 100 cm marks respectively on a metre rule whose weight is negligible.
   What is the magnitude and location of the single upward force which balances the system?

3. A tapered pole, which is 5.00 m long and weighs 400 N, can be balanced at a point 2.00 m from the thicker end. If it were to be supported at its ends, how much force would be needed at each end?

4. A telegraph pole is placed on a two-wheeled dolly located 4.00 m from the thicker end and an upward force of 1500 N at the thinner end is required to keep it horizontal. The pole is 15.0 m long and weighs 9000 N. Where is the centre of gravity?

5. A 600 N bricklayer is 1.50 m from one end of a uniform 800 N scaffold which is 7.00 m long. A pile of bricks weighing 500 N is 3.00 m from the same end. If the scaffold is supported at the two ends, calculate the force on each support.

6. A uniform beam 4.00 m long weighs 300 N and is supported at its ends by two walls. Find the reaction of the walls against the beam when a man weighing 750 N stands on the beam at a distance of 1.20 m from one end.

7. A uniform horizontal bar 2.00 m long weighs 250 N. Upward forces of 150 and 100N are exerted at the ends.
   a) Is the bar in equilibrium?
   Calculate:
   b) the torque about each end.
   c) the torque about the centre.
   d) the torque about a point 1.50 m from one end of the bar.

8. A uniform rod 120 cm long has a mass of 1.20 kg. It has a mass of 5 .00 kg attached to it at one end, a mass of 4.00 kg at the other end and a mass of 2.00 kg in the middle.
   Find the position of the centre of gravity.

9. A wheel of 0.25 m diameter has an axle of 3.00 cm radius. If a force of 300 N is exerted along the rim of the wheel, what is the smallest force exerted on the outside of the axle which will result in zero net torque?

10. Two vehicles are crossing a bridge 20.0 m long. A passenger car weighing 12 000 N is 6.00 m from one end. A truck weighing 40 000 N is 7.00 m from the opposite end. If the bridge is symmetrical with respect to the centre and weighs 1.20 MN, what are the forces on the two supports at the ends of the bridge?

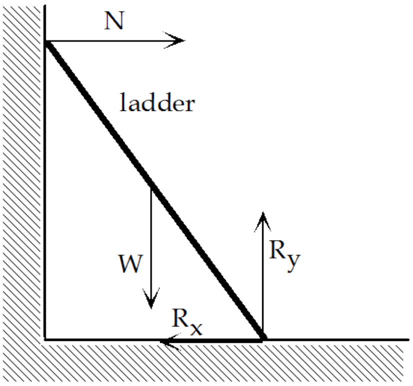
ANSWERS

1. a) 14 kN b) 1.2 m
2. 26.0 N at 64.4 cm
3. 240 N; 160 N
4. 5.83 m from thick end
5. 1157 and 743 N
6. 675 N; 375 N
7. a) No b, c, d) 50 Nm in each case
8. 55.1 cm from 5 kg mass
9. 1250 N
10. 622 kN; 630 kN

Moments Applications Involving Non-Parallel Forces

1. Ladder problem

A ladder may be considered as a uniform beam with the weight of the ladder (W) acting through its mid point. Reaction forces are exerted on the ladder at each end where the ladder makes contact with the wall and the ground. Note we are only interested in the forces acting on the ladder.



When a ladder rests against a **smooth** vertical wall, we assume the surface can only provide a **normal reaction** force, N, to the surface. That is, the reaction force is at right angles to the surface.

A smooth surface cannot provide a sideways frictional force.
This force acts on the upper most end of the ladder at right angles to the wall.

The ground provides the necessary reaction (R_Y) to balance the weight of the ladder and also provides a sideways component (R_X) to balance the normal reaction (N).

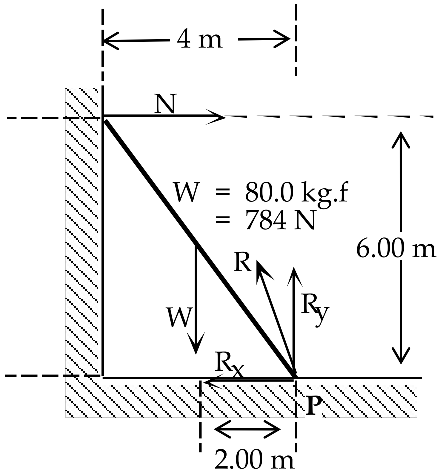
The total reaction force provided by the ground is the vector sum of these two components (R_X and R_Y).

Example 1: Simple Ladder Problem

One edge of a uniform ladder of mass 80.0 kg rests against a smooth wall at a point 6.00 m above the ground.

The other end is on the ground, at P, 4.00 m from the wall.

Find the magnitude and direction of the reaction of the ground on the ladder.



Solution:

A. Find components of R (i.e. R_X and R_Y)

Take moments about P:

Σ CM $=$ Σ ACM ;

$\therefore$ $6 \times N = 2 \times W$

$\therefore$ $N = \frac{2W}{6} = \frac{2(784)}{6} = \underline{261.3\text{ N}}$

Now $\mathbf{R_X = N = \underline{261\text{ N}}}$ **and** $\mathbf{R_Y = W = \underline{784\text{ N}}}$

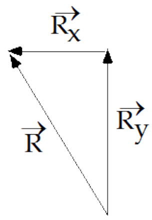
B. Find magnitude of R

$$\vec{R} = \vec{R}_X + \vec{R}_Y$$

$$R^2 = R_x^{\,2} + R_y^{\,2}$$

$\therefore$ $R^2 = 784^2 + 261^2$

$\therefore$ $R = \underline{826\text{ N}}$



C. Find direction of R

$$\tan\theta = \frac{R_x}{R_y} = \frac{261.3}{784}$$

$\therefore$ $\mathbf{\underline{\theta = 18.4^o}}$

D. Answer: $\vec{R} =$ **826 N**

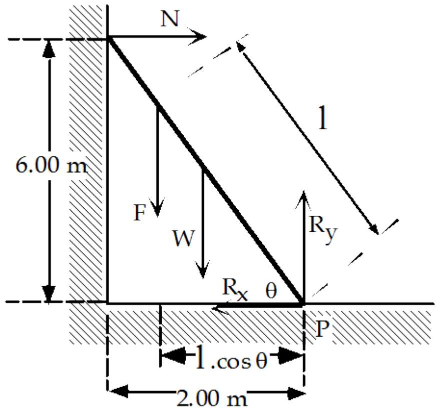
at 18.4^o to the vertical.

Example 2: Harder Ladder Problem

A ladder weighs 1.50 x 10^2 N and rests against a smooth wall 6.00 m above the ground.

The base of the ladder is 2.00 m from the wall.

How far up the ladder can a man weighing 750.0 N climb if the maximum frictional force between the ladder and the ground is 1.80 x 10^2 N ?



Solution:

There is no frictional force between the ladder and the wall.

The force of friction with the ground is equivalent to R_X.

We assume the ladder is uniform, so its weight will act half way along the ladder.

Let W and F be the weight of the ladder and the weight of the man respectively.

Let l be the distance up the ladder the man climbs.

$W = 1.50 \times 10^2$ N

$F_W = 750.0$ N

$F_f = R_X = 1.80 \times 10^2$ N

$F_{left} = F_{right}$

$\therefore N = R_X = 1.80 \times 10^2$ N

$\tan\theta = \frac{6}{2} \quad \therefore \theta = 71.57^0$

Taking moments about P:

Σ ACM $= \Sigma$ CM

$(l \times \cos\theta) \times F + 1 \times W = 6 \times N$

$\therefore (l \times \cos\theta) \times F = 6N - W$

$$\therefore l = \frac{(6 \times N - W)}{\cos\theta \times F}$$

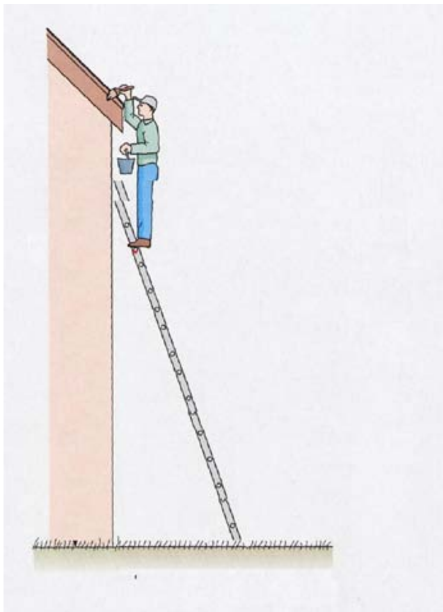
$$= \frac{6(180) - 150}{\cos 71.57\ (750)}$$

$\therefore l =$ **<u>3.92 m</u>**

Problem: Consider a man climbing up a ladder up a smooth wall
Carefully describe and explain what happens to the

a) reaction force of the wall on the ladder

b) frictional force of the ground on the ladder



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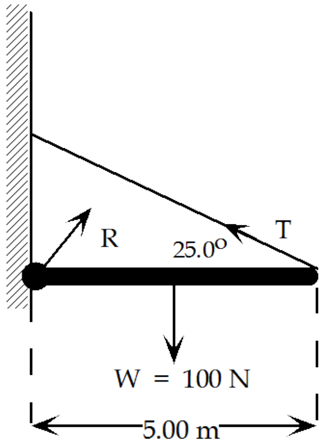
Example 3: Simple Cantilever Problem

A uniform plank is 5.00 m long and is hinged at one end to a wall.

A rope connects the other end to the wall and makes an angle of 25.0^0 to the horizontal.

If the plank is held horizontal and weighs 1.00×10^2 N, find the tension in the rope and the total reaction at the hinge.



Suggested approach:

1. Find all forces
2. $\sum F = 0$ (i.e. Co-Planar forces in equilibrium)
3. Solve

1. Find Forces

Taking moments about hinge:

$$\sum CM = \sum ACM$$

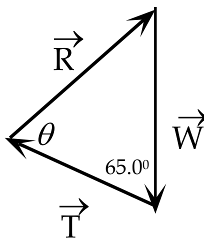
$$5 \times T \sin 25 = 2.5 \times 100$$

$$T = \frac{100 \times 2.5}{5 \times \sin 25}$$

$$T = \underline{\mathbf{1.18 \times 10^2}}\ \mathbf{N}$$

2. Sum all forces to find R

$$\sum \vec{F} = 0 \quad \therefore \vec{W} + \vec{T} + \vec{R} = 0$$



$$R^2 = T^2 + W^2 - 2W\,T \cos 65$$

$$R^2 = 118.3^2 + 100^2 - 2(100)(118.3) \cos 65$$

$$\mathbf{R = \underline{118.3}\ N}$$

3. Find θ

$$\frac{\sin\theta}{W} = \frac{\sin 65}{R}$$

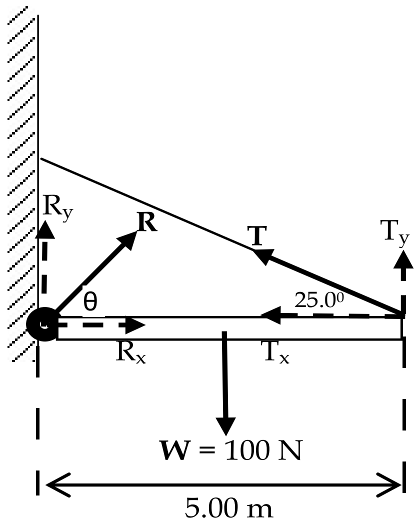
$$\sin\theta = \frac{W \sin 65}{R} = \frac{100 \sin 65}{118.3}$$

$$\mathbf{\theta = \underline{50.0^o}}$$

Answer: $\vec{R} = $ **118 N at 25.0^o to the horizontal.**

Note that moments were taken about the hinge so that the equation would have only one unknown. Moments could have been taken about other points but this would be less convenient.

Alternative method to Cantilever problems



1. Use an x-y axes system and resolve all vectors into their x and y components.
2. Then solve for $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

The free-body diagram showing all the forces acting on the plank has been redrawn below with all x and y components included.

Step 1: Use Moments

This is exactly the same as the previous method.

Taking moments about hinge:

$\Sigma CM = \Sigma ACM$

$5 \times T \sin 25 = 2.5 \times 100$

$T = \dfrac{100 \times 2.5}{5 \times \sin 25}$

$T = \underline{1.18 \times 10^2}$ **N**

Note, again, how important it is to be careful in choosing the pivot point.

Step 2: Find Equations using $\Sigma F = 0$

The sum of the forces in the horizontal (x) direction:

$\Sigma F_X = 0$

$R_X - T_X = 0$ i.e. $R_X = T_X$ but $T_X = T \cos 25^0$

So, $R_X = T \times \cos 25^0$

$= 118.3 \times \cos 25^0$

Therefore $R_X = 107.2$ N

The sum of the forces in the vertical (y) direction:

$\Sigma F_Y = 0$

$R_Y + T_Y - 100 = 0$

So, $R_Y + T \times \sin 25^0 = 100$

$R_Y + 118.3 \times \sin 25^0 = 100$

$R_Y + 50.0 = 100$

Therefore $R_Y = 50.0$ N

Step 3: Use Pythagoras to solve for Reaction

$$R = \sqrt{R_X^2 + R_Y^2} = \sqrt{107.2^2 + 50.0^2} = 1.18 \times 10^2 \text{ N}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{50.0}{107.2} \qquad \theta = 25.0^0$$

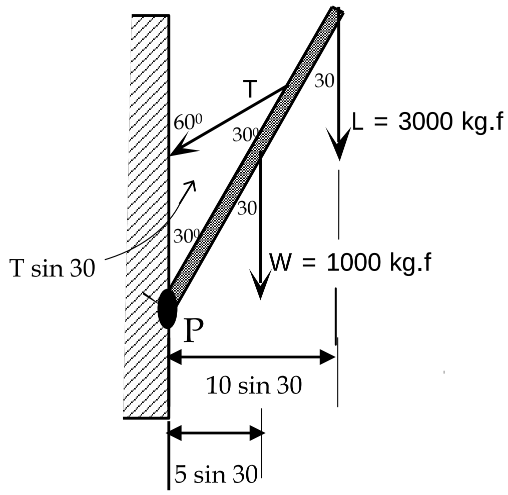
Answer: $R = 1.18 \times 10^2$ N at 25.0^0 to the horizontal.

Example 4: The Jib Problem

The next example involves a uniform rod that is attached to a wall by a hinge and is supported by a flexible cable. It is important to note that a flexible cable can support a force only along its length.

(If there were a component of force perpendicular to the cable, it would bend because it is flexible).

But for a rigid device, such as the hinge, the force can be in any direction and we can know the direction only after solving the problem.



This means the reaction force R will probably not lie along the rod but will instead be at some angle to the rod.

A jib and tie arrangement (with load, L) is shown in the diagram.

The jib is 10.0 m long and is attached to the wall at the hinge P.

The mass of the jib is 1.00×10^3 kg.

The tie is a flexible cable attached 3.00 m from the end of the jib.

The load has a mass of 3.00×10^3 kg.

Find the tension, T, in the cable and the reaction, R, which the hinge exerts on the rod.

Suggested Strategy:

Step 1: Use moments about the hinge P to find the tension, T.

Step 2: Use Equilibrium, i.e. $\sum F = 0$, to find the magnitude of the reaction force R.

Step 3: Use trigonometry to find the angle of the reaction force.

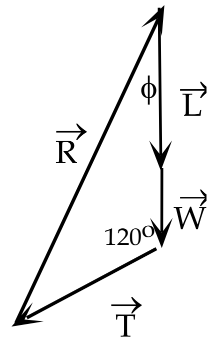
Step 1: $\sum ACM = \sum CM$

$$7 \times T \sin 30^0 = W \times 5 \times \sin 30^0 + L \times 10 \times \sin 30^0$$

$$= 1000 \times 5 \times \sin 30^0 + 3000 \times 10 \times \sin 30^0$$

$$T = \frac{5000\sin 30 + 30000\sin 30}{7\sin 30} = 5000 \text{ kgf}$$

$$= \mathbf{4.90 \times 10^4\ N}$$



Step 2: $\sum \vec{F} = 0 \quad \therefore \vec{W} + \vec{L} + \vec{T} + \vec{R} = 0$

Now use the Cosine Rule: $R^2 = T^2 + (W+L)^2 - 2(W+L) \times T \cos 120^0$

$$= 5000^2 + 4000^2 - 2(4000) \times (5000)\cos 120$$

R = 7810 kgf

Step 3: Find angle ϕ $\quad \dfrac{\sin\phi}{T} = \dfrac{\sin 120^0}{R} \qquad \sin\phi = \dfrac{T \times \sin 120^0}{R} = \dfrac{5000 \sin 120}{7810}$

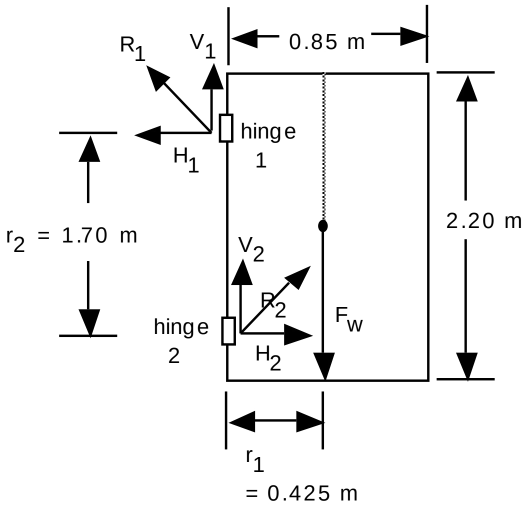
$$\phi = 33.7^0$$

Answer: $\vec{R}$ **= 7810 kgf at 33.7° to the vertical.**

Example 5: Door Problem

The door shown has a mass of 35.0 kg and is hanging from both its hinges.

If each hinge carries an equal share of the load, find the magnitude and direction of the force at each hinge.



Reasoning:

If $V_1 = V_2 = V$ and $V_1 + V_2 = F_W$

then $V = \frac{F_W}{2} = \frac{35.0 \times 9.80}{2}$

i.e. $V_1 = V_2 =$ **171.5 N**

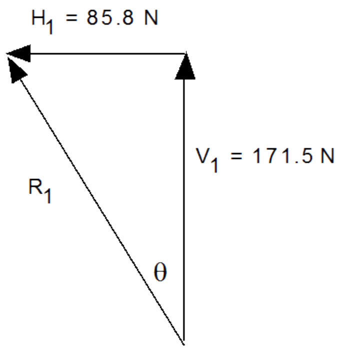
Solution:

Taking moments about H_2: Σ CM = Σ ACM

$r_1 \times F_W = r_2 \times H_1$

$H_1 = \frac{0.425 \times 35.0 \times 9.80}{1.70}$

$H_1 = H_2 =$ **85.8 N**



Reaction at H_1 $R_1^2 = H_1^2 + V_1^2$

$R_1^2 = 85.8^2 + 171.5^2$

i.e. $R_1 = 192$ N

and $\tan\theta = \frac{85.8}{171.5} \Rightarrow \theta =$ **26.6°**

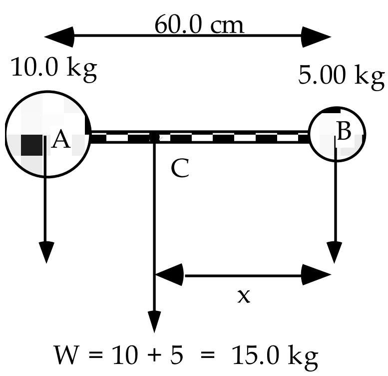
i.e. **R_1 = 192 N at 26.6° to the vertical towards the wall**.

Reaction at H_2: Since $V_1 = V_2$ and $H_2 = H_1$

Then **R_2 = 192 N at 26.6° to the vertical away the wall.**

Example 6: Finding the Centre of Gravity

Two spheres of mass 5.00 kg and 10.0 kg are connected by a bar of negligible mass. If the distance between the centres of the two spheres is 60.0 cm, find the position of the centre of mass of the combination.



Taking moments about B (centre of mass):

$(x)\,W_T = 60\,W_A$

$x = \frac{60\,W_A}{W_T} = \frac{(60)\,10}{15}$

$=$ 40.0 cm

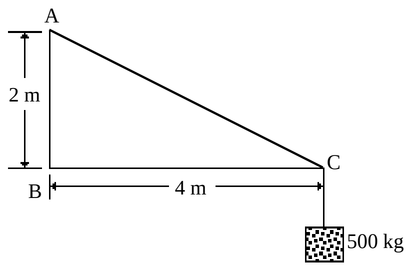
∴ **C is 40.0 cm from 5.00 kg sphere.**

Exercise Set 4: Moments –Non parallel forces

1. A uniform bar, 2.00 m long has a mass of 25.0 kg. It is hinged at one end and is kept in a horizontal position by a rope attached to the free end, making an angle of 45.0^0 with the bar and in the same vertical plane as the bar. If 60.0 kg is hung on the bar 1.20 m from the hinge, what is the tension in the rope?

2. A wall crane has the dimensions shown in the diagram and a mass of 500 kg is suspended at C.

   Determine the forces in AC and BC and state whether they are tensile or compressive.



3. A uniform beam weighing 1.00×10^3 N is pivoted at one end and held in a horizontal position by a rope from the free end to a point vertically above the pivot. Use the general conditions of equilibrium to find the tension in the rope and the reaction at the pivot when the rope is inclined at 45.0^0 to the beam.

4. A uniform rod is 1.80 m long and weighs 50.0 N. Its lower end rests on level ground to which it is inclined at an angle of 60.0^0 and it rests against a smooth rail at a point 60.0 cm from its upper end. Find the magnitude and direction of the reaction at the ground.

5. A garden roller has a radius of 30.0 cm, weighs 500 N and stands on level ground against a step 6.00 cm high.
   a) What horizontal force must be exerted to make the roller rise off the ground?
   b) What is the least force necessary to do this?

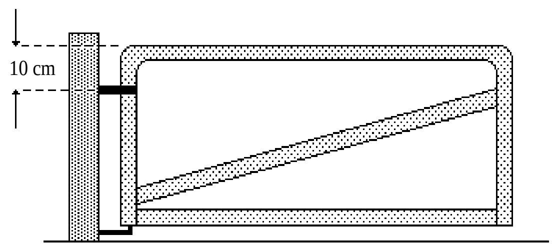
6. A street lamp weighing 25 N is suspended above the roadway from one end of a uniform rod which is hinged to a vertical post at the other end. The rod, which weighs 50 N and is 1.8 m long, is held in a horizontal position by means of a flexible wire attached 60 cm from the lamp. The other end of the wire is attached to the post so that the wire makes an angle of 60^0 with the post. Find the tension in the wire.

7. A 2.50 m uniform ladder weighing 200.0 N rests on rough ground and against a smooth wall 2.00 m above ground level. Determine the magnitude and direction of the reaction at the ground when a 780.0 N man stands 50.0 cm from the top of the ladder.

8. A farm gate as shown is 4.00 m long and 1.20 m high and is uniformly constructed such that its weight of 185 kg acts through its central point.

   The lower hinge supports the entire weight of the gate.

   Determine the magnitude and direction of the reaction at each hinge.



9. A ladder is 8.00 m long, has its centre of gravity 3.00 m from the bottom and weighs 240 N. A girl weighing 400 N stands halfway up the ladder, which makes an angle of 20.0^0 with the vertical.
   a) Find the force exerted on the ladder by the smooth wall.
   b) Find the horizontal and vertical components of the force exerted on the ladder by the ground.

10. A uniform ladder 5.00 m long weighs 200 N. It leans against a smooth wall, making an angle of 60.0° with the horizontal ground. How far up the ladder can a man weighing 700 N climb before the ladder slips if the maximum frictional force the ground can provide is 400 N?

11. A 240 N door 2.50 m high and 1.20 m wide is hung by two hinges, each of which supports half the weight. If the hinges are 0.30 m from the top and 0.30 m from the bottom of the door, find the horizontal components of the forces exerted on the door by the hinges.
[Assume that the centre of gravity of the door is at the geometrical centre]

12. A uniform 200 N gate is 1.60 m high and 1.20 wide. It is supported by two hinges, the upper of which is 0.10 m from the top and bears three-fifths the weight. The lower hinge is 0.10 m from the bottom. Find the horizontal and vertical components of the force exerted on the gate by the lower hinge.

13. A uniform girder is carried horizontally on the shoulders of two men, Eric and Fred. The girder is 4.00 m long and weighs 8.00×10^2 N. Eric, who carries half the load Fred carries, is at the extreme end of the girder. How far from the other end is Fred?

14. A painter works on a horizontal platform that is suspended from two ropes 2.00 m apart. The tension in one rope is 3.00×10^2 N and in the other is 4.00×10^2 N.
The painter weighs 6.00×10^2 N. Where is the painter standing on the platform?

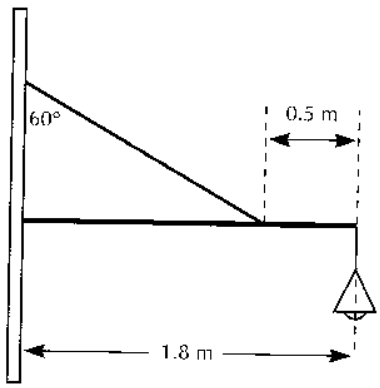
15. A 5.00×10^2 N girl exerts a force equal to a quarter of her weight as she pushes in turn, on each pedal of her bike. The arm of the pedal is 0.300 m long.
   a) Where will the pedals be when she is generating the maximum torque?
   b) How great is this maximum torque?
   c) If the arm of the pedal were longer, she could generate a greater torque.
   Why aren't bicycles made with longer arms attached to their pedals?

16. A street lamp weighing 20.0 N is suspended above the roadway from one end of a horizontal rod, 1.80 m long, which is fastened at its other end to a vertical post.

The rod weighs 30.0 N. It and the lamp are partly supported by a very light cable, attached 50.0 end is fastened.

a) Calculate the tension in the cable.

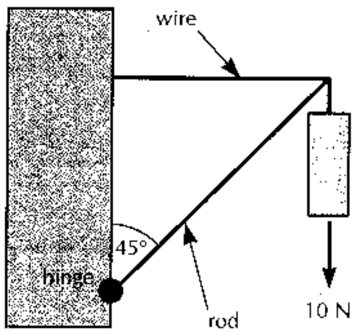
b) Calculate the direction and magnitude of the force the rod exerts on the post.



17. A strong, light rod is pivoted against a firm wall and its free end is supported by a horizontal wire, also attached to the wall.

The rod makes an angle of 45.0° with the wall when a 10.0 N weight is hung from the end to which the wire is attached.

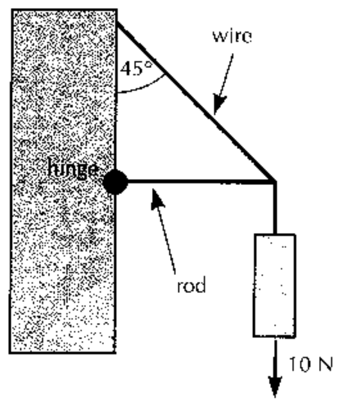
Calculate the forces in the rod and the wire.



18. A strong, light rod is hinged to a firm wall and held horizontally by a wire, attached between its free end and the wall.

The rod makes an angle of 45° with the wire and the wall.

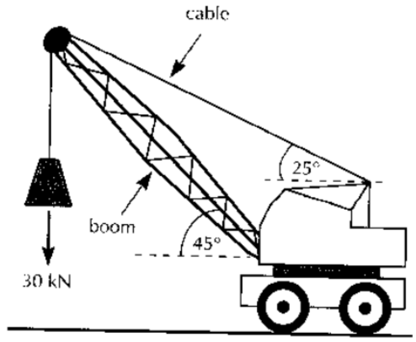
Calculate the forces in the rod and the wire when a 10 N load is hung from the point where they are joined.



19. Explain why a chair can be pushed across the floor by pushing at the top of its back, but an equal force applied at the same place in the opposite direction may tip the chair over.

20. A crane lifts a load of 30 kN with a cable and a boom, as shown in the diagram.

Calculate the forces in the cable and in the boom.

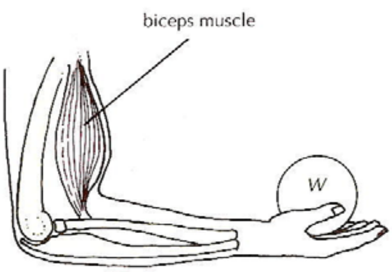


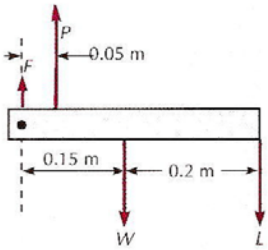
Answers

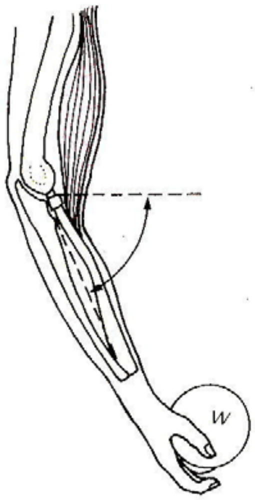
1. 673 N
2. 11.0 kN tension in AC, 9.8 kN compression in BC.
3. 7.07×10^2 N; 7.07×10^2 N at an angle of 45^o with the beam.
4. 43.8 N at an angle of 68.2^o with the horizontal.
5. a) 375 N b) 300 N
6. 150 N
7. 1.13 kN at 60.3^o above the horizontal.
8. 3.30 kN horizontally, 3.76 kN at 28.8^o to horizontal.
9. a) 106 N b) 106 N; 640 N
10. 4.23 m
11. 75.8 N for both hinges
12. 85.7 N; 80.0 N
13. 1.0 m
14. 0.83 m from 400 N rope
15. b) 37.5 Nm
16. a) 97 N, b) 84 N at 1^0 to the rod
17. 10 N
18. 14N
20. 62.0 kN, 79.5 kN

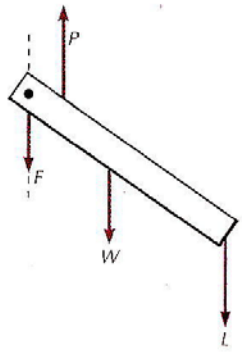
Biomechanical Examples

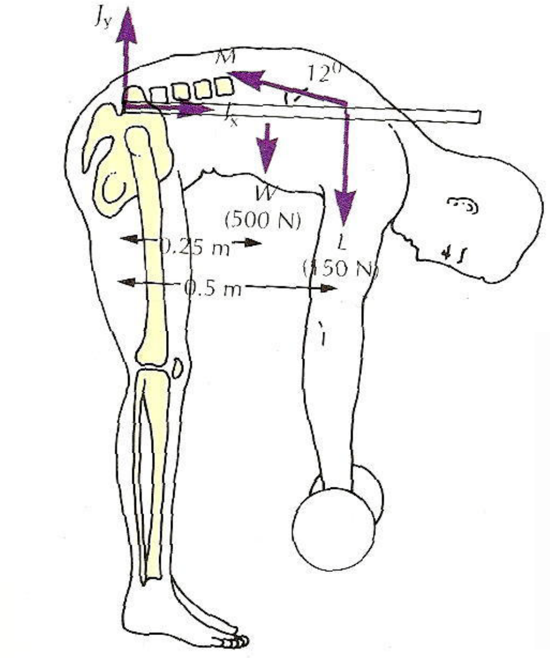
The principles of mechanical equilibrium can also be applied to biomechanical contexts. The strategies for problem-solving are identical and the spatial arrangements are generally simplified with schematic representations.

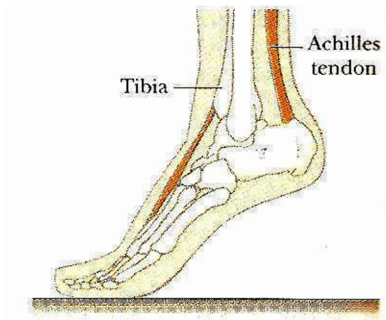


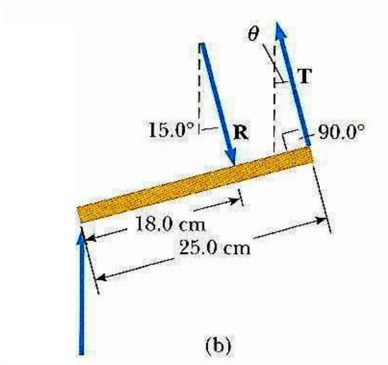




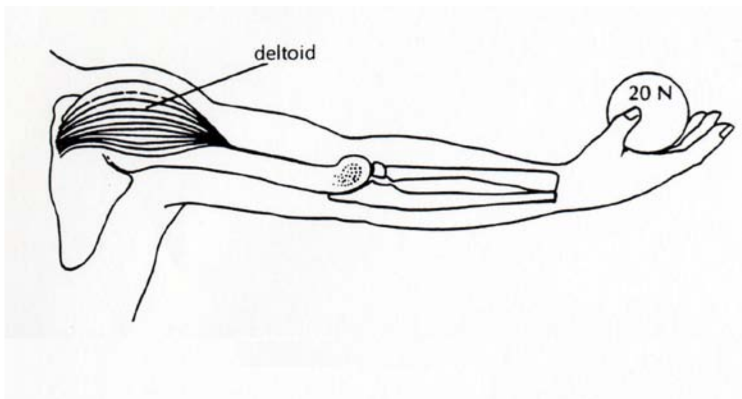








Exercise 1 –Biomechanical Shoulder



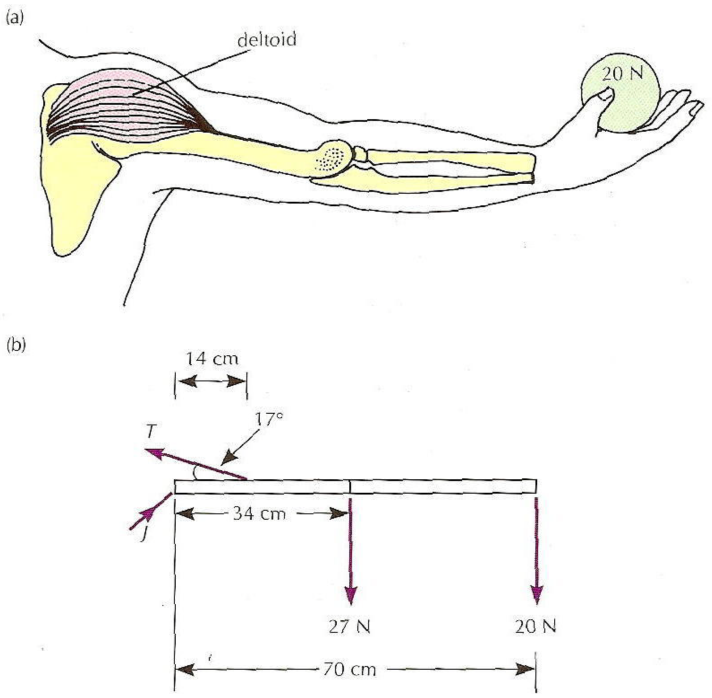
Consider a the tension in the deltoid muscle :

a) Draw a free body diagram to identify the major forces that enables you to hold a mass in an outstretched hand. (on the given diagram).

b) Summarise this information by constructing a simple schematic (below the diagram).

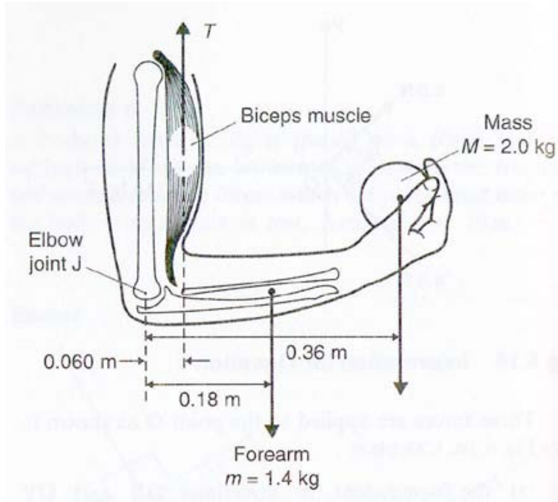
Problem 1 –Biomechanical Shoulder

Given the information provided in the diagrams, determine the tension in the deltoid muscle.



Problem 2 –Biomechanical Biceps

Given the information provided in the diagrams determine the tension in the biceps muscle and the reaction at the elbow.



Static Equilibrium and Structures

When a body or system is not accelerating or rotating, it is in both translational and rotational equilibrium (i.e. $\Sigma F = 0$ and $\Sigma M = 0$), and so it is said to be in static equilibrium.

A building should be in static equilibrium.

Exercise: Read the relevant pages from your textbook (Cahill) and summarise the important key points relating to the following structures:

Cantilevers:

Struts and Ties:

Arches: